Quarkonium and Lattice Gauge Theory

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Outline

1. Basics
2. Lattice Quarkonium
3. Lattice Stoponium
4. Summary

what we want is

• we would like to calculate low energy non-perturbative quantities in QCD reliably and quantitatively.

• with the systematic errors (from finite lattice spacing, finite spacetime volume, finite quark mass) under control

• cf. K.G. Wilson, PRD10 (1974) 2445

and where we are for some quantity

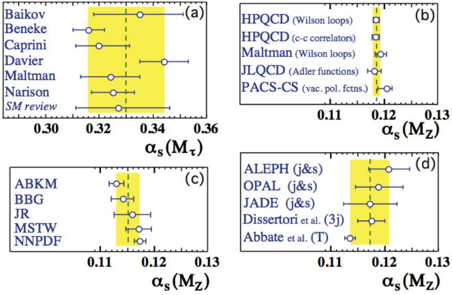


Figure 9.2: Summary of determinations of α_s from hadronic τ-decays (a), from lattice calculations (b), from DIS structure functions (c) and from event shapes and jet production in e^+e^--annihilation (d). The shaded bands indicate the average values chosen to be included in the determination of the new world average of α_s.

2012 PDG summary on QCD

lattice QCD

- K.G. Wilson, PRD10 (1974) 2445
- non-perturbative regularization of quantum field theory
- discretization of the Euclidean spacetime

  $\rightarrow$ give up the continuous spacetime symmetry
- keep the gauge symmetry
- quantum physics is "a numerical integral problem" according to Feynman's path integral formulation
- Monte Carlo method for the numerical integral

lattice QCD

- Schroedinger equation

$$i\,\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}\psi \tag{1}$$

- path integral

$$\langle O\rangle = \frac{\int [dq]\; O\;\; e^{i\int dt\mathcal{L}}}{\int [dq]\; e^{i\int dt\mathcal{L}}} \tag{2}$$

Wick rotation ($t \to i\tau$)

$$\langle O\rangle = \frac{\int [dq]\; O\;\; e^{-\int d\tau\mathcal{L}}}{\int [dq]\; e^{-\int d\tau\mathcal{L}}} \tag{3}$$

lattice QCD

- path integral formulation of quantum field theory

$$\langle \mathcal{O} \rangle = \frac{\int dA_\mu d\psi d\overline{\psi} e^{-S_E} \mathcal{O}[A_\mu, \psi, \overline{\psi}]}{\int dA_\mu d\psi d\overline{\psi} e^{-S_E}} \tag{1}$$

- infinite dimensional integral problem
- Monte Carlo method for numerical integral
- importance sampling

$$\langle \mathcal{O} \rangle \sim \frac{1}{N} \sum_i \mathcal{O}[A_i] \tag{2}$$

lattice QCD

- caveats :

  fermion doubling problem

  integration of Grassmann variables

  breaking of continuous spacetime symmetry

lattice QCD

- numerically (largest problem I am involved with) :

$48^3 \times 64$ lattice ($\sim$ 4 GBytes per gauge configuration)

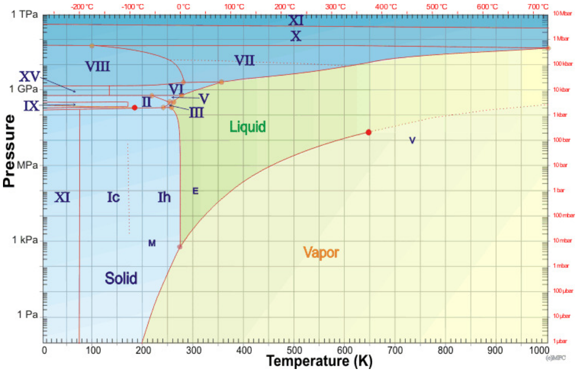
$N \sim 400$

no of $\beta's \sim 10$

takes a few year to generate Monte Carlo samples of gauge configurations on one of the fastest supercomputers

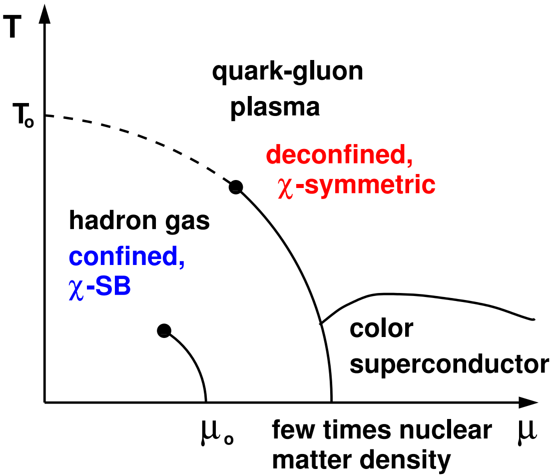
QCD phase diagram

QCD phase diagram

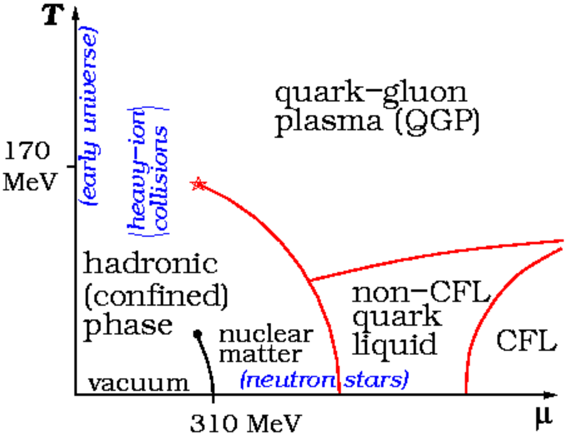


from Wikipedia

QCD phase diagram

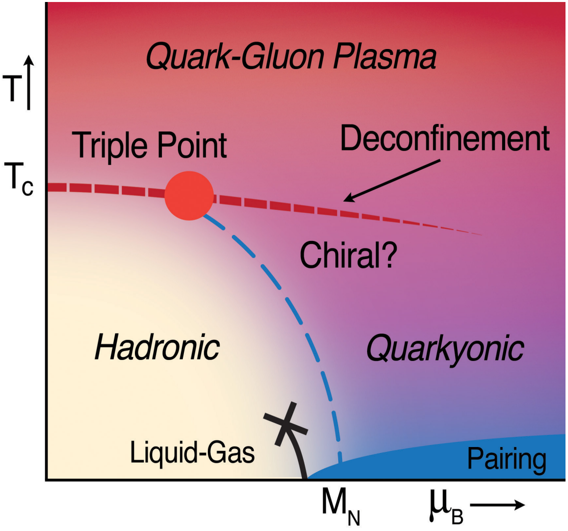


QCD phase diagram



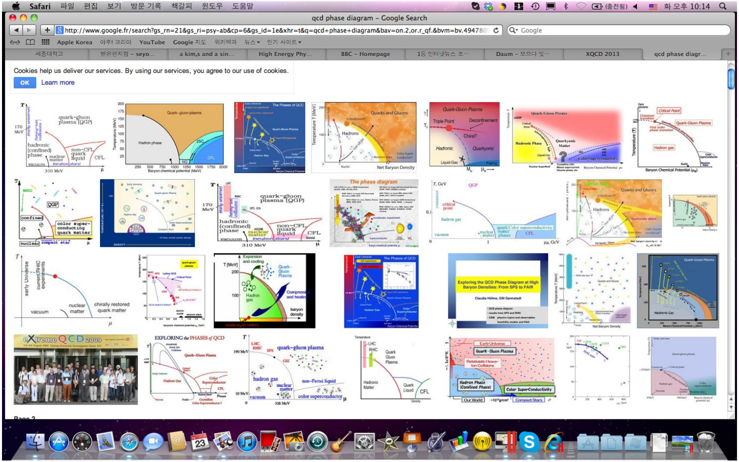
from Wikipedia

QCD phase diagram

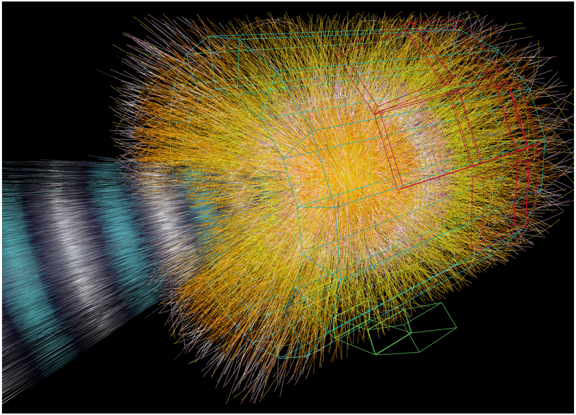


from arXiv:0911.4806

QCD phase diagram



QCD phase diagram



Motivation

- quarkonium, a “thermometer” for QGP

- M_b is quite larger than “binding energy”

- use lattice NRQCD to separate M_b scale physics from bound state scale

- lattice NRQCD bottomonium correlator + reconstruction of spectral function on non-zero temperature

Motivation

- expect no new UV complication at non-zero T
- temperature, $T = \frac{1}{N_\tau a_\tau}$
- for consistent lattice NRQCD,

$$Ma_\tau \sim 1 \rightarrow N_\tau \sim \frac{M}{T}$$

- for $M = M_b(\sim 4.5\mathrm{GeV})$ and $T \sim 2T_c(\sim 2 \times 150\mathrm{MeV})$

$$N_\tau = O(10)$$

- at $T > 0$, spectral function is more complicated while available lattice data points is smaller ($\rightarrow$ reliability of the method is important)

spectral function in NRQCD

In QCD,

$$G_\Gamma(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Gamma\psi(0,\vec{0})\rangle \quad (1)$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\vec{p}) \quad (2)$$

and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \quad (3)$$

• the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

• numerically ill-posed problem

• Maximum Entropy Method is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)

spectral function in NRQCD

- known to have problems (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)

- both the kernel($K(\tau,\omega)$) and the spectral density($\rho_\Gamma(\omega,\vec{p})$) depend on temperature

- constant contribution

- In NRQCD, with $\omega = 2M + \omega'$ and $T/M << 1$, $K(\tau,\omega) \to e^{-\omega\tau}$

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \qquad (1)$$

- numerical inverse Laplace transform problem

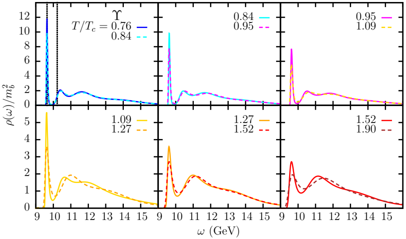
previously

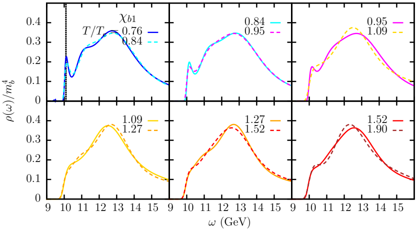
FASTSUM collaboration
(Frascati-Argonne-Swansea-Trinity-Sejong-Utah-Maynooth, G. Aarts, C. Allton, S.K., B. Oktay, S. Ryan, D.K. Sinclair, J.-I. Skullerud, and +)

- bottomonium NRQCD correlator analysis (PRL106 (2011) 061602)
- S-wave bottomonium spectral function at rest (JHEP1111(2011) 103)
- S-wave bottomonium spectral function moving (JHEP1303(2013) 084)
- P-wave bottomonium spectral function at rest (JHEP1312(2013) 064)

previously

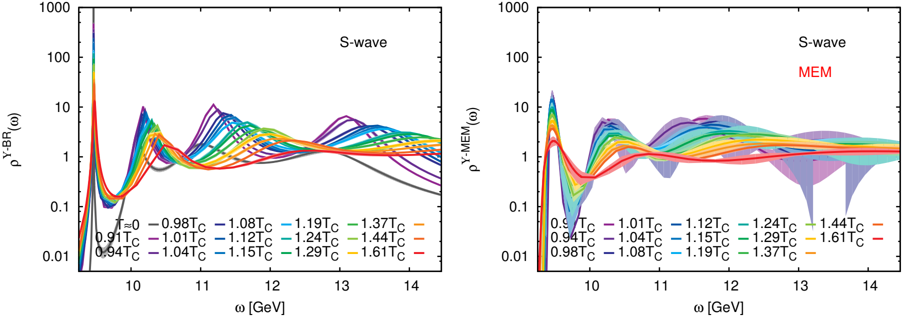
- Bottomonium, FASTSUM with $N_f = 2+1$ (JHEP1407 (2014) 097)





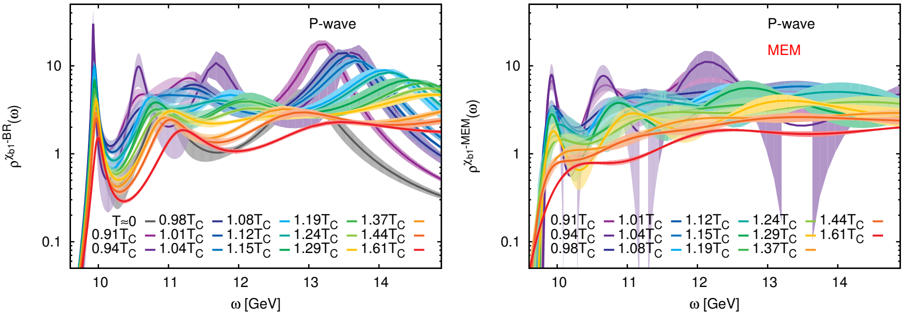
previously

- Bottomonium, S.K., P. Petreczky, A. Rothkopf with $N_f = 2 + 1$ (PRD91 (2015) 054511)



previously

- Bottomonium, KPR with $N_f = 2 + 1$ (PRD91 (2015) 054511)



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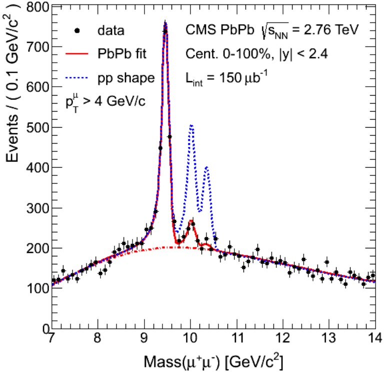


- CMS collaboration, PRL107 (2011) 052302

Charmonium at $T \neq 0$

- can we see more pronounced temperature effect in charmonium?
- melting at lower temperature?

Charmonium at $T \neq 0$

- $\mathcal{O}(v^2)$: ~ 0.3 for J/ψ and ~ 0.1 for Υ
- $\mathcal{O}(a^2 p^2)$ and $\mathcal{O}(aK)$
- radiative corrections in the coefficients of NRQCD lagrangian

lattices ($N_f = 2+1$ HotQCD configurations with HiSQ)

$M_b a$	$M_c a$	β	T(MeV)	T/T_c	a_τ^{-1} (fm)	No. of Conf.(analyz
2.759	0.7566	6.664	140.40	0.911	0.1169	400
2.667	0.7314	6.700	145.32	0.944	0.1130	400
2.566	0.7035	6.740	150.97	0.980	0.1087	400
2.495	0.6841	6.770	155.33	1.008	0.1057	400
2.424	0.6657	6.800	159.33	1.038	0.1027	400
2.335	0.6403	6.840	165.95	1.078	0.09893	400
2.249	0.6167	6.880	172.30	1.119	0.09528	400
2.187	0.5996	6.910	177.21	1.151	0.09264	400
2.107	0.5776	6.950	183.94	1.194	0.08925	400
2.030	0.5566	6.990	190.89	1.240	0.08600	400
1.956	0.5364	7.030	198.08	1.286	0.08288	400
1.835	0.5030	7.100	211.23	1.371	0.07772	400
1.753	0.4806	7.150	221.08	1.436	0.07426	400
1.559	0.4274	7.280	248.63	1.614	0.06603	400

Table: summary for the $T \neq 0, 48^3 \times 12$ lattice data set

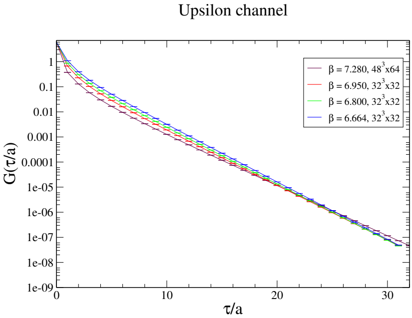
lattices ($N_f = 2 + 1$ HotQCD configurations with HiSQ)

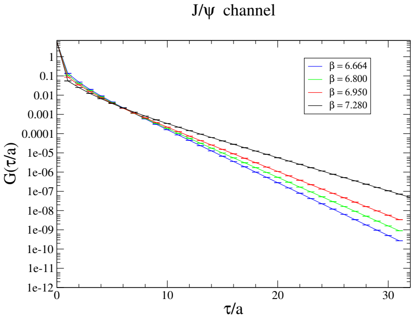
N_s	N_t	β	T(MeV)	T/T_c	a_τ^{-1} (fm)	No. of Conf.(analyzed)
32	32	6.664	140.40	0.911	0.1169	1600
48	48	6.740	150.97	0.980	0.1087	400
32	32	6.800	159.33	1.038	0.1027	400
48	48	6.880	172.30	1.119	0.09528	400
32	32	6.950	183.94	1.194	0.08925	400
48	48	7.030	198.08	1.286	0.08288	400
48	64	7.150	221.08	1.436	0.07426	400
48	64	7.280	248.63	1.614	0.06603	400

Table: summary for the $T = 0$ lattice data set

$T = 0$ comparison (preliminary)

- $G(\tau)$ for 3S1 bottomonium (left) and charmonium (right)





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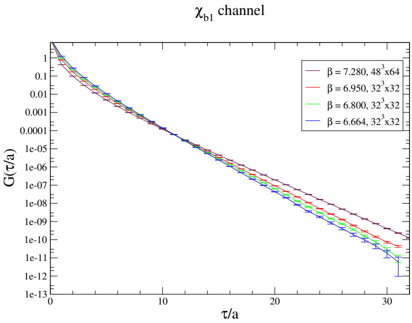
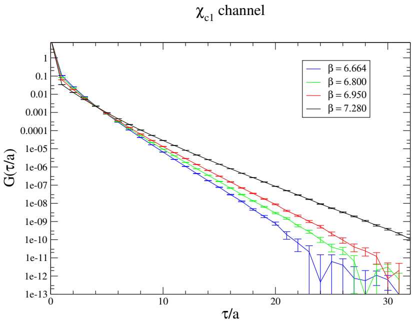
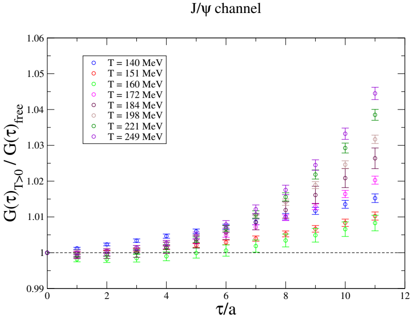
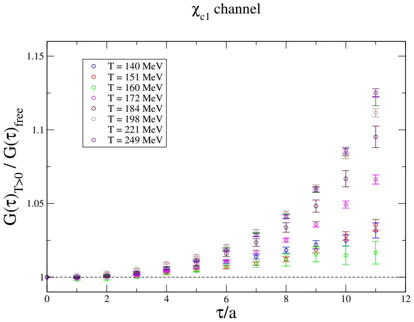




$T \neq 0$ correlator analysis (preliminary)

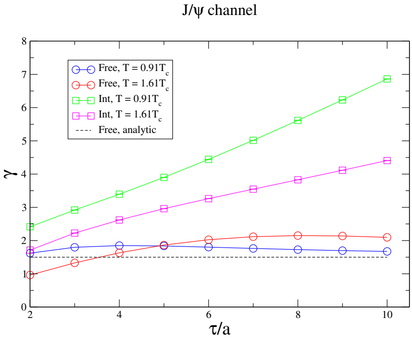
- $\frac{G_{T>0}(\tau)}{G_{T\simeq 0}(\tau)}$ for several β

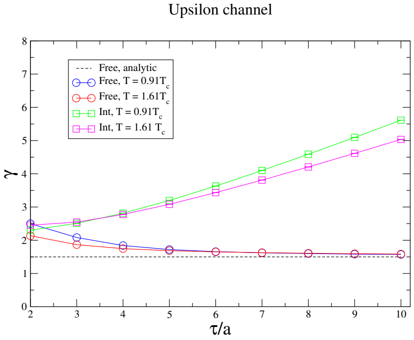




$T \neq 0$ correlator analysis (preliminary)

- $\gamma = \dfrac{\frac{\tau dG(\tau)}{d\tau}}{G(\tau)}$ (if free, $G(\tau) \sim A\tau^{\gamma}$ and $\frac{\tau dG(\tau)}{d\tau} / G(\tau) \sim \gamma$)
(cf. PRL106 (2011) 061602)





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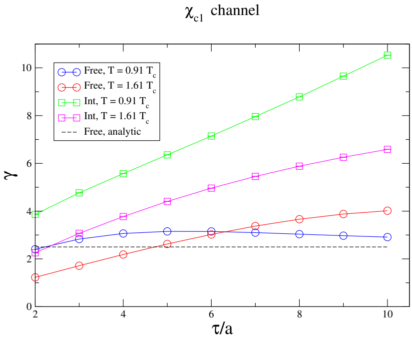
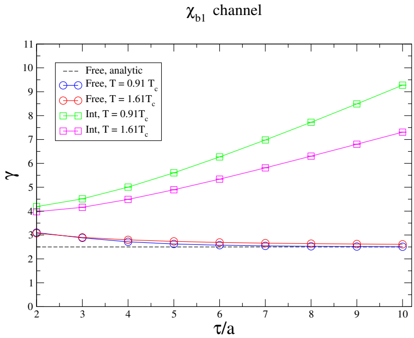
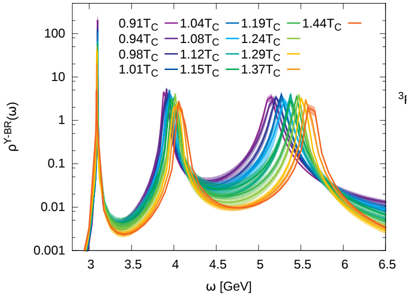
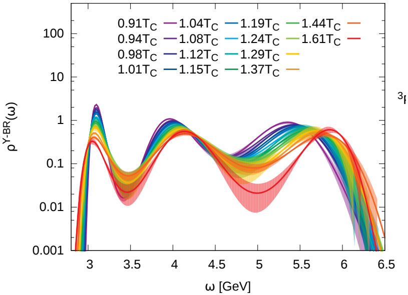




$T \neq 0$ spectral functions (preliminary)

- 3S1 charmonium spectral function from BR (left) and MEM (right)





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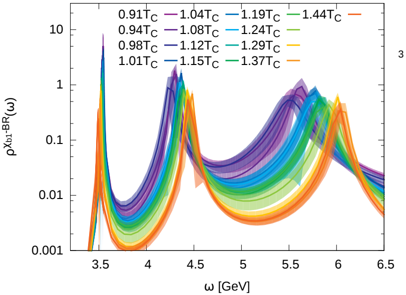
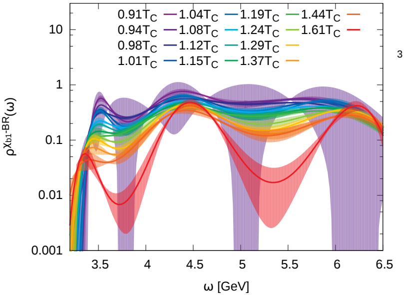




Summary – quarkonium

- Despite the large relativistic effect, a tentative investigation on charmonium at non-zero temperature is under way.

- Larger temperature effect is seen for charmonium system than bottomonium system

- S-wave $T \neq 0$ charmonium correlators show binding behavior upto $T = 1.61\,T_c(= 249)$ MeV

- P-wave $T \neq 0$ charmonium correlators show free behavior at $T = 1.61\,T_c(= 249)$ MeV

- Further work on Bayesian reconstruction of spectral function will follow

stopnonium

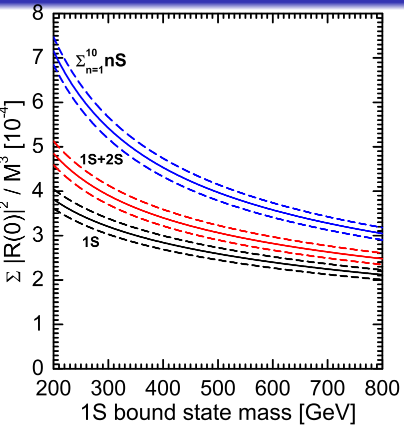
• bound states of heavy scalar quark ($m \sim m_t$) under the strong interaction

• active searches at LHC experiment (e.g., N.Kumar and S.P. Martin, PRD 90, 055007 (2014), recently, B. Batell and S. Jung, arXiv:1504.01740)

• computation of production/decay rates requires

$$\frac{|\phi(0)|^2}{M^3} \tag{2}$$

stopnonium



- $\frac{|\phi(0)|^2}{M^3}$ from “potential model” (cf. K. Hagiwara, K. Kato, A.D. Martin, and C.-K. Ng, NPB344, 1 (1990))

stopnonium

• consistency condition for quarkonium bound states

$$Mv^2 \sim \frac{\alpha_s(1/r)}{r}, \quad r \sim \frac{1}{Mv} \tag{2}$$

$$\alpha_s(Mv) \sim v \tag{3}$$

• for example, naive expectation of "Coulombic bound state" doesn't work and running of α_s makes the short distance behavior softer (cf. K. Hagiwara, K. Kato, A.D. Martin, and C.-K. Ng, NPB344, 1 (1990))

non-relativistic QCD

- non-relativistic effective theory of QCD doesn't require modeling of the potential

- scale separation: M, Mv, Mv^2

- Non-relativistic QCD in FT

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}, \tag{4}$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi, \tag{5}$$

and

non-relativistic QCD

$$\begin{aligned}
\delta\mathcal{L} = \quad & -\frac{c_1}{8M^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi-\chi^\dagger(\mathbf{D}^2)^2\chi\right] \\
& +c_2\frac{ig}{8M^2}\left[\psi^\dagger\left(\mathbf{D}\cdot\mathbf{E}-\mathbf{E}\cdot\mathbf{D}\right)\psi+\chi^\dagger\left(\mathbf{D}\cdot\mathbf{E}-\mathbf{E}\cdot\mathbf{D}\right)\chi\right] \\
& -c_3\frac{g}{8M^2}\left[\psi^\dagger\sigma\cdot\left(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D}\right)\psi+\chi^\dagger\sigma\cdot\left(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D}\right)\chi\right] \\
& -c_4\frac{g}{2M}\left[\psi^\dagger\sigma\cdot\mathbf{B}\psi-\chi^\dagger\sigma\cdot\mathbf{B}\chi\right]. \qquad (4)
\end{aligned}$$

- the last two lines are absent in the case of stoponium

lattice related

• $16^3 \times 256$ "quenched" lattices

• for stoponium, confinement is not important (size is smaller than ~ 1 fm)

• $\beta = 6/g^2 = 8.751$ ($a^{-1} = 50$ GeV, from S.K. D.K. Sinclair, PRD48 4408 (1993))

• $\mathcal{O}(v^2)$ NR lagrangian

• NR-related unknown energy shift E_0 and mass renormalization : mean field approximation

$$M_n = 2(Z_M M - E_0) + E_n \tag{5}$$

lattice related

- energy shift :

$$E_0 = -a^{-1} \ln \left[u_0 (1 - a h_0/(2n))^{2n} \right] \tag{5}$$

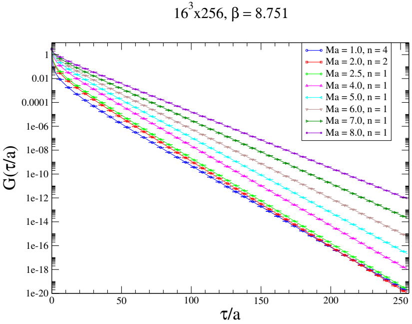
- mass renormalization :

$$Z_M = u_0^{-1} (1 - a h_0/(2n)) \tag{6}$$

where $u_0 (= \langle 0 | \frac{1}{3} \mathrm{Tr} U_{plaq} | 0 \rangle^{1/4})$ is the tadpole improvement factor, and

$h_0 = 3(1 - u_0)/Ma^2$

S-wave stoponium



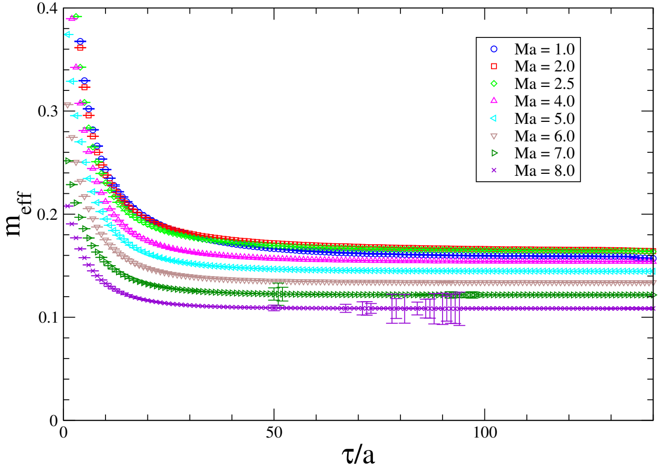
- correlator

$$G(\tau) = \sum_n e^{-E_n \tau} |\langle 0|\phi(0)|n\rangle|^2 \tag{7}$$

if the states are well defined stationary states,

$$\rightarrow G(\tau) \sim a_0 e^{-E_0\tau} + a_1 e^{-E_1\tau} + a_2 e^{-E_2\tau} + \cdots \tag{8}$$

S-wave stoponium



- effective mass

S-wave stoponium

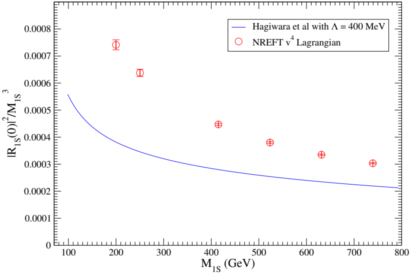
Ma	Z_M	$E_0 a$	$2(Z_M M - E_0)a$	$E_{1S}a$	$\lvert\phi(0)\rvert^2 a^{-3/2}$
1.0	1.047	0.2852	1.524	0.1391(10)	0.00337(8)
2.0	1.047	0.1788	3.830	0.1665(4)	0.0113(2)
2.5	1.030	0.1583	4.834	0.1718(3)	0.0191(3)
4.0	1.047	0.1257	8.125	0.1746(1)	0.0610(4)
5.0	1.053	0.1149	10.30	0.1721(1)	0.1041(7)
6.0	1.056	0.1078	12.46	0.1685(1)	0.1611(8)
7.0	1.059	0.1027	14.62	0.1643(1)	0.2341(12)

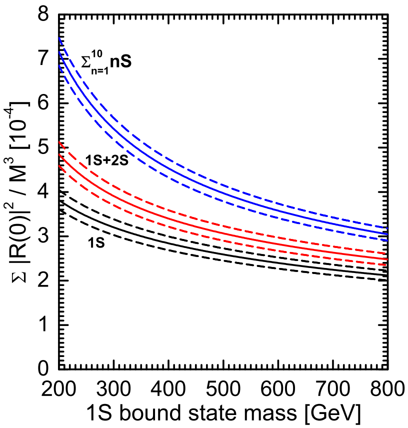
Table: lattice result at $\beta = 8.751$

Ma	M_{1S} (GeV)	$\lvert\phi(0)\rvert^2 / M_{1S}^3 [\times 10^{-4}]$
1.0	83.14(5)	30.7(14)
2.0	199.84(2)	7.41(18)
2.5	250.28(5)	6.38(1)
4.0	414.96(1)	4.47(4)
5.0	523.44(1)	3.80(3)
6.0	631.49(1)	3.35(2)
7.0	[illegible]	[illegible]

S-wave stoponium

- comparison: $\frac{|\phi(0)|^2}{M^3}$ from lattice vs. potential model





Summary – stoponium

- stoponium, bound states of heavy scalar quark and anti-scalar quark, which are binded by the strong interaction are studied with lattice gauge theory method

- $\frac{|\phi(0)|^2}{M^3}$: factor 1.5 $\sim$ 2 larger than a potential model result

- further study on systematic errors is necessary

  quenched approximation $\rightarrow$ dynamical quarks

  finite volume

  mean field approximation

  relativistic correction

  radiative correction

  and etc

Summary

• some non-perturbative quantities in the strong interaction can be calculated quantitatively and reliably by lattice gauge theory

• some aspects of the strong interaction can be calculated qualitatively by lattice gauge theory

• with improvements in algorithm and computing power, continued progresses are expected

• lattice calculations on light hadron spectrum, quarkonia, B meson physics, K meson physics, $g-2$, $\cdots$ become accurate enough to have impacts on phenomenology.

• cf. annual lattice conference proceedings.