Breaking discrete symmetries in the effective field theory of inflation

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Based on D. Cannone, JG and G. Tasinato, to appear in JCAP [arXiv:1505.05773 [hep-th]]

Outline

Why effective field theory?

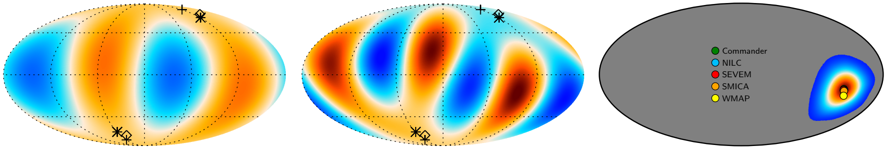
Inflation seems to dwell in *terra incognita*

- $E_{\rm inf} \sim 10^{16}$ GeV (?) $\gg E_{\rm LHC} = 14 - 15$ TeV
- Are various corrections important or not?
- Control over our ignorance: universality of EFT

Basic symmetry principles in the EFT of inflation are (Cheung et al. 2008)

1. Spatial diffeomorphism is invariant (homog & iso)
2. Time translational symmetry is broken (t-dep BG)

Are these symmetries all?



- Tantalizing observational hints
  1. Quadrapole and octopole orientations are misaligned $(9-13^\circ)$
  2. Preferred dipole direction: $A = 0.07$ and $(l, b) = (227^\circ, -15^\circ)$
- Theoretical motivations
  1. Direction-dependent ingredient, e.g. vector field
  2. Models with special conditions: solid / elastic inflation...

Why broken discrete symmetries?

(Slighly) broken spatial diff may be allowed (Cannone, Tasinato & Wands 2015) but studying a system that breaks all diff during inflation is difficult

- We study *discrete* symmetries
- Parity violation: a lot of studies
- Time reversal: nothing to the best of our knowledge
- We adopt model-independent EFT approach to study the consequences of leading operators

Comoving (unitary) gauge

Single field case: for a single scalar field $\phi(t,\boldsymbol{x}) = \phi_0(t) + \delta\phi(t,\boldsymbol{x})$

$$x^\mu \to x^\mu + \xi^\mu(t,\boldsymbol{x}) \quad \text{then} \quad \delta\phi \to \delta\phi - \dot{\phi}_0\xi^0$$

By setting $\delta\phi = 0$, all dynamical d.o.f. are given to metric

“Comoving (unitary)” gauge

Inflation by a set of fields which we collect within a 4-vector Ψ^μ

$$\delta\Psi^\mu \to \delta\Psi^\mu - \dot{\Psi}^\mu{}_{,\nu}\xi^\nu$$

$\delta\Psi^\mu = 0$: generalized unitary gauge, and only metric perts are left

BG with anisotropies

BG metric = homogeneous FRW + anisotropic parts

$$\bar{g}_{\mu\nu} = \bar{g}^{(0)}_{\mu\nu} + \bar{g}^{(a)}_{\mu\nu} = a^2(\eta)\begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix} + a^2(\eta)\begin{pmatrix} & \beta_i \\ \beta_i & \chi_{ij} \end{pmatrix}$$

To support anisotropies, EMT contains θ_i and σ_{ij}:

$$S_m = -\int d^4x\sqrt{-g}\left[\Lambda(\eta) + c_1(\eta)g^{00} + c_2(\eta)\delta_{ij}g^{ij} + d_1(\eta)\theta_i g^{0i} + d_2(\eta)\sigma_{ij}g^{ij}\right]$$

Einstein eqs give relations bet coeffs and aniso parameters

$$\begin{aligned} 3\mathcal{H}^2 &= c_1 + 3c_2 + a^2\Lambda \\ \mathcal{H}^2 - \mathcal{H}' &= c_1 + c_2 \\ d_1\theta_i &= c_2\beta_i \\ 2d_2\sigma_{ij} &= \mathcal{H}\chi'_{ij} + \frac{1}{2}\chi''_{ij} + 2c_1\chi_{ij} \end{aligned}$$

Reduction to isotropic universe

To reduce our labour on huge possibilities of ops

1. Residual symmetry: $x^i \to x^i + \xi^i(t)$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2(\eta) h_{\mu\nu} \quad \text{with} \quad h_{0i} = S_i + \partial_i B$$

$$\text{This gives} \quad S_i \to S_i - \partial_\eta \xi_i^T \quad \text{and} \quad B \to B - \partial_\eta \xi^L + \xi^0$$

   h_{0i} is eliminated unless there is spatial derivs acting on it

$$d_1 = 0 \quad \text{thus} \quad \beta^i = 0$$

2. Shear is set to vanish: $\sigma_{ij} = 0$ so that we focus on θ_i
   The source for χ_{ij} is vanishing, so simply we choose $\chi_{ij} = 0$

$$\text{The BG metric is isotropic: } \bar{g}_{\mu\nu} = a^2(\eta) \begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix}$$

Mass terms

Basic strategies

1. Due to residual symmetry, h_{0i} does not show up
2. No floating spatial indices

For those without derivative, out of 3 [h_{00}, h_{ii}, h_{ij} $(i \neq j)$] we can construct 4 quadratic operators:

1. $m_1^2 h_{ij}^2 = m_1^2 \left[12\varphi^2 + 2(\partial_i F_j)^2 + \gamma_{ij}^2 + 8\varphi\nabla^2 E + 4(\nabla^2 E)^2 \right]$
2. $m_2^2 h_{ii}^2 = m_2^2 \left(6\varphi + 2\nabla^2 E \right)^2$
3. $m_3^2 h_{00}^2 = m_3^2 4A^2$
4. $m_4^2 h_{00} h_{ii} = m_4^2 \left(12A\varphi + 4A\nabla^2 E \right)$

Single-derivative operators

$$
\begin{aligned}
\mathcal{O}_1^{(1)} &= \mu a^3 \epsilon_{ijk} (\partial_i h_{jm}) h_{km} = \mu a^3 \epsilon_{ijk} \left[(\partial_i \gamma_{jm}) \gamma_{km} - \partial_i F_j \Delta F_k \right] \\
\mathcal{O}_2^{(1)} &= \mu a^3 \epsilon_{ijk} \theta_i h_{jm} h'_{km} \\
&= \mu a^3 \epsilon_{ijk} \theta_i \big(\gamma_{jm} \gamma'_{km} - F_m \partial_j \gamma'_{km} - F'_m \partial_k \gamma_{jm} \\
&\qquad\qquad - F_j \Delta F'_k + 2 F'_{k,j} \Delta E + 2 F_{k,j} \Delta E' \big)
\end{aligned}
$$

1. $\mathcal{O}_1^{(1)}$ leads to parity violation: not inv under $x^i \to -x^i$
2. $\mathcal{O}_2^{(1)}$ breaks time-reversal

Double-derivative operators

$$
\begin{aligned}
\mathcal{O}_1^{(2)} &= a^2 h'_{ij}\theta_j\partial_k h_{ik} \\
&= -a^2\theta_j\Big(4\varphi'\partial_j\varphi + 2\varphi'\Delta F_j + \gamma'_{ij}\Delta F_i - 2\varphi\Delta F'_j - 2F'_j\Delta^2 E \\
&\qquad +4\varphi'\partial_j\Delta E + 4\partial_j\varphi\Delta E' + 4\Delta E'\partial_j\Delta E - F'_i\partial_j\Delta F_i\Big) \\
\mathcal{O}_2^{(2} &= a^2 h'_{ij}\theta_k\partial_k h_{ij} \\
&= -a^2\theta_k\Big(12\varphi'\partial_k\varphi + \gamma'_{ij}\partial_k\gamma_{ij} \\
&\qquad +4\varphi'\partial_k\Delta E + 4\partial_k\varphi\Delta E' + 4\Delta E'\partial_k\Delta E - F'_i\partial_k\Delta F_i\Big)
\end{aligned}
$$

Not inv under an indep interchange of time and spatial coords

No propagating vector modes

We are interested in pert linear in θ_i

$$\mathcal{L}^{(v)} = \underbrace{\frac{a^2}{2}\left[\partial_k\left(S_i - F_i'\right)\partial_k\left(S_i - F_i'\right)\right]}_{\text{EH part}} \underbrace{-2m_1^2 a^4 \left(\partial_i F_j\right)^2}_{\text{mass term } m_1^2} \underbrace{-\theta_i F_j(\cdots)}_{\text{everything else}}$$

S_i is not dynamical and can be solved, then the eq for F_i reads

$$\Delta F_i = \frac{\theta_i}{m_1^2}(\cdots)$$

Thus overall $\mathcal{L}^{(v)} = \mathcal{O}(\theta_i^2)$: no propagating vector modes

New operator for the scalar sector

Considering $\mathscr{O}_i^{(2)}$ the new contribution to the scalar Lagrangian is

$$\mathscr{L}^{(s)} \supset a^2 \varphi' \theta_i \partial_i \varphi$$

With the standard procedure $u = z\varphi$ ($z \propto a$ during dS)

$$S^{(s)} = \int d^4x \frac{1}{2}\left[u'^2 - (\nabla u)^2 + \frac{z''}{z}u^2 + 2b_1\theta_i u' \partial_i u\right]$$

(b_1: real and constant for simplicity)

$$u_k'' + 2ib_1\theta_i k_i u_k' + \left(k^2 - \frac{z''}{z}\right)u_k = 0$$

Direction-dependent phase in the scalar sector

We can discover the standard mode function equation by writing

$$u_k = \exp\left(-ib_1\theta_i k_i \eta\right) u_k^{(0)}$$

which gives

$$u_k^{(0)\prime\prime} + \left(k^2 - \frac{z''}{z}\right) u_k^{(0)} + \underbrace{k^2\left(b_1\theta_i\hat{k}_i\right)^2 u_k^{(0)}}_{\mathcal{O}(\theta_i^2)\text{ so neglected}} = 0$$

We find a direction-dependent phase which keeps $\mathcal{P}_{\mathcal{R}}$ isotropic

c.f. In the configuration space, $u(\eta, x^i) = u^{(0)}(\eta, x^i + b_1\theta^i\eta)$

New operators in the tensor sector

We consider $\mathcal{O}_i^{(1)}$ $\left[\text{c.f. } \mathcal{O}_i^{(2)} \text{ the same as the scalar}\right]$

$$S^{(t)} = \int d^4x \frac{a^2}{8}\left[{\gamma'_{ij}}^2 - (\nabla\gamma_{ij})^2 + 2q_1\mu a\epsilon_{ijk}(\partial_i\gamma_{jm})\gamma_{km} + 2q_2\mu a\epsilon_{ijk}\theta_i\gamma_{jm}\gamma'_{km}\right]$$

For each circular pol component λ (+=right, −=left)

$$\gamma''_{(\lambda)} + 2\mathcal{H}\gamma'_{(\lambda)} + k^2\gamma_{(\lambda)} - 2\lambda q_1\mu ak\gamma_{(\lambda)} - 2\lambda q_2\mu a\hat{\theta}\gamma'_{(\lambda)} - 3\lambda q_2\mu a\mathcal{H}\hat{\theta}\gamma_{(\lambda)} = 0$$

where $\hat{\theta} = i\theta_i\hat{k}_i$

- q_1 term: associated with spatial deriv, well-known parity violating ops
- q_2 terms: associated with time deriv on $a\gamma_{(\lambda)}$, novel!

Chiral phase in the tensor sector

Defining

$$v_{(\lambda)} \equiv \frac{a}{\sqrt{2}}\gamma_{(\lambda)} = \exp\left(i\lambda q_2\mu\theta_i\hat{k}_i\int a d\eta\right) v_{(\lambda)}^{(0)}$$

we recover the standard tensor mode eq at linear order in θ_i:

$$v_{(\lambda)}^{(0)\,\prime\prime} + \left(k^2 - \frac{a''}{a}\right) v_{(\lambda)}^{(0)} = 0$$

Noticing that $\int a d\eta \approx N_e/H$, the number of e-folds

$$\gamma_{(\lambda)} = \exp\left(i\lambda q_2\mu\theta_i\hat{k}_i\frac{N_e}{H}\right)\gamma_{(\lambda)}^{(0)}$$

Phase modulation but it dep on a) chirality and b) number of e-folds

c.f. In the configuration space $\gamma_{(\lambda)}(\eta, x^i) = \gamma_{(\lambda)}^{(0)}\left(\eta, x^i - \lambda q_2\mu\frac{N_e}{H}\theta^i\right)$

Conclusions

- Effective field theory of inflation
  - Conventionally only time translational symmetry is broken
  - Tantalizing hints for broken spatial diffeomorphism inv
- Anisotropic metric and energy-momentum tensor
  - In general we have pure vector $(0i)$ and pure tensor (ij)
  - We concentrate on the effects of vector θ_i
- Novel quadratic operators
  - Imposing residual symmetry is powerful
  - Up to 2 derivatives
- Dynamics of fluctuations
  - Vector: no propagating physical d.o.f.
  - Scalar: direction-dependent phase
  - Tensor: phase that dep on chirality and number of *e*-folds