Towards realistic models in SO(32) heterotic string theory

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1. Introduction
2. Torus compactification with magnetic fluxes
3. SO(32) heterotic string theory
4. Gauge coupling unification
5. Summary

based on

H.Abe, T.K., H.Otsuka, Y.Takano, 1503.06770

H.Abe, T.K., H.Otsuka, Y.Takano, T.H.Tatsuishi, 1507.xxxxx

1 Introduction

Superstring theory is a promising candidate
for unified theory of
all interactions including gravity,
(gauge bosons and graviton),
and all matter fields,
quarks and leptons and higgs fields.

Theory of Everything

Our world

If superstring theory is really the theory of everything and/or it is meaningful in particle physics and cosmology,

from superstring theory one can derive

the standard model (at low-energy scale)

SU(3)xSU(2)XU(1) gauge symmetry,

three chiral generations of quarks and leptons as well as higgs,

experimental values, gauge couplings, Yukaws,

and answer further mysteries in particle physics and cosmology, e.g. DM, inflaton, etc.

Superstring phenomenology and cosmology

String phenomenology

Superstring : theory around the Planck scale

SUSY ? GUT ?

? ? ? ? ? Several scenario ? ? ? ? ? ?

Standard Model : we know it up to 100 GeV

Superstring → low energy ? （top-down）

Low energy → underlying theory ? （bottom-up）

Both approaches are necessary to

connect between underlying theory and our Nature.

String phenomenology

Superstring predicts extra 6D compact space

in addition to our 4D spacetime.

There are lots of compactifications,

i.e. string landspace,

and physics depends on compactifications

There is no stringy principle to select one (for now).

However, we don’t need to care about the compactifications, which can not realize our world,

electron mass = 0.5 MeV,

fine structure constant = 137,

etc.

Torus with magnetic flux

The SM is a chiral theory,
left-handed and right-handed fermions have
SU(3)xSU(2)xU(1) charges different from each other.

torus compactification is simple,
but leads to non-chiral theory (as shown later).

Torus with magnetic fluxes leads to
a chiral theory.

Torus with magnetic flux

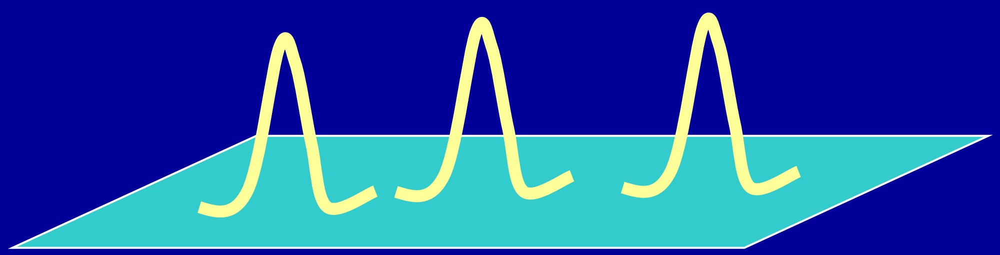
The number of zero-modes,

i.e. the generation number,

is determined by magnetic flux.

Magnetic flux

$\Rightarrow$ non-trival zero-mode profile

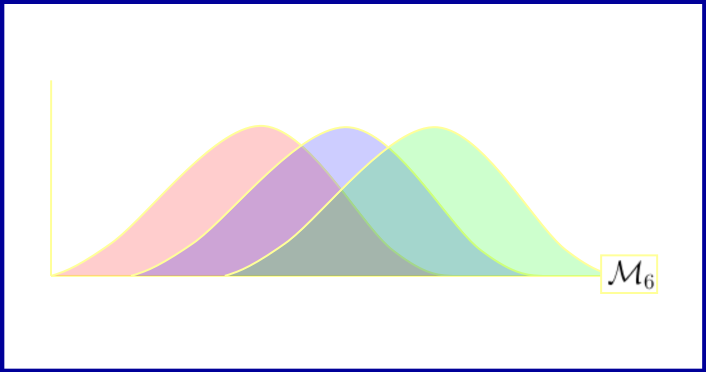


Yukawa couplings

The Yukawa couplings are obtained by overlap integral of their zero-mode w.f.'s.

$$Y = g\int d^2 z\, \psi^i_M(z)\psi^j_N(z)\psi^k_P(z)$$

$$z = y_4 + iy_5$$



$\Rightarrow$ O(1) coupling

suppressed coupling

Torus with magnetic flux

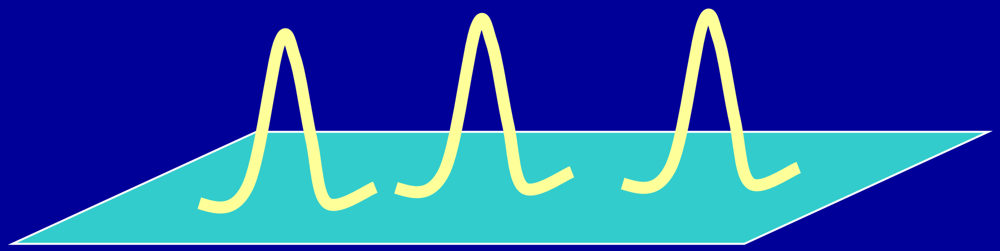
Magnetic flux is simple,

but interesting.

chiral theory

the generation number

non-trivial profile



2. Torus with magnetic flux

2-1. Extra dimensions

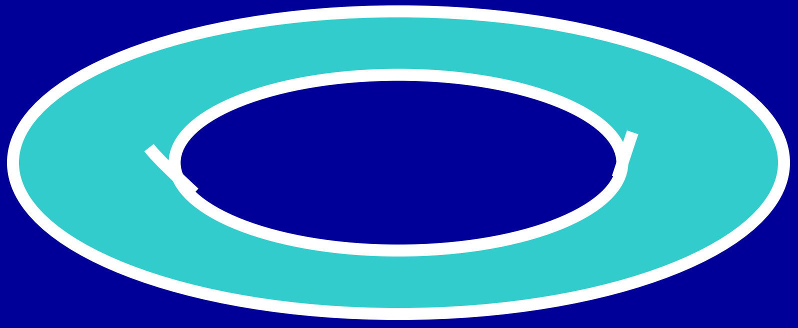
4 + n dimensions

4D $\Rightarrow$ our 4D space-time

nD $\Rightarrow$ compact space

Examples of compact space

torus, orbifold, CY, etc.



Field theory in higher dimensions

10D $\Rightarrow$ 4D our space-time + 6D space

10D vector

$$\boxed{A_M \Rightarrow A_\mu , A_m}$$

4D vector + 4D scalars

SO(10) spinor $\Rightarrow$ SO(4) spinor

x SO(6) spinor

internal quantum number

Several Fields in higher dimensions

4D (Dirac) spinor

$$i\gamma^{\mu} D_{\mu}\psi = 0$$

$\Rightarrow$ (4D) Clifford algebra $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$

(4x4) gamma matrices

represention space $\Rightarrow$ spinor representation

6D Clifford algebra $\{\gamma^{M}, \gamma^{N}\} = 2\eta^{MN}$

$$\begin{array}{ll} \gamma^{M}\ (M = 0,1,2,3) & \gamma^{\mu} \otimes \sigma^{3} \\ \gamma^{4},\ \gamma^{5} & I_{4\times 4} \otimes \sigma^{1},\quad I_{4\times 4} \otimes \sigma^{2} \end{array}$$

6D spinor $\psi_{4D} \otimes \psi_{2D}$

6D spinor $\Rightarrow$ 4D spinor x (internal spinor)

internal quantum number

Field theory in higher dimensions

Mode expansions

$$\partial^M \partial_M A_m = (\partial^\mu \partial_\mu + \Delta_6) A_m = 0$$

$$i(\Gamma^\mu D_\mu + \Gamma^m D_m)\lambda = 0$$

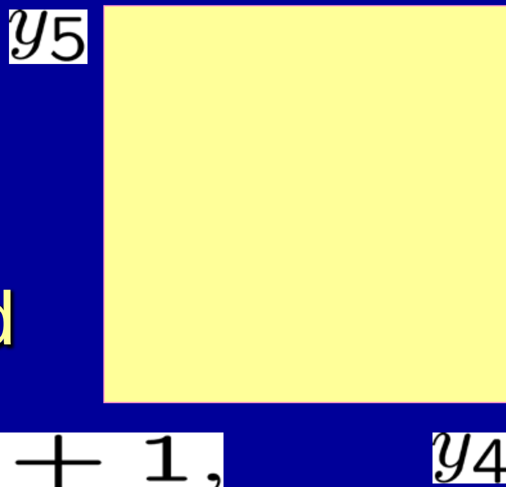
$$\begin{aligned}
\lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\
A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m)
\end{aligned}$$

$$\begin{aligned}
i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\
\Delta_6 \phi_{n,M}(y) &= M^2_{n,M} \phi_{n,M}
\end{aligned}$$

KK decomposition

KK docomposition on torus

$$
\begin{aligned}
i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\
\Delta_6 \phi_{n,M}(y) &= M_{n,M}^2 \phi_{n,M}
\end{aligned}
$$



torus with vanishing gauge background

Boundary conditions

$$
\begin{aligned}
y_4 &\sim y_4 + 1, \\
y_5 &\sim y_5 + 1
\end{aligned}
$$

$$
\begin{aligned}
\phi(y_4+1, y_5) &= \phi(y_4, y_5) \\
\phi(y_4, y_5+1) &= \phi(y_4, y_5)
\end{aligned}
$$

$$
\begin{aligned}
&\phi_0 : \text{constant mode} && m_n = 0 \\
&\phi_n : \exp(ikny) \quad k = 2\pi / R && m_n = kn
\end{aligned}
$$

We concentrate on zero-modes.

4D effective theory

Higher dimensional Lagrangian (e.g. 10D)

$$L_{10} = g\int d^4x d^6y \, \overline{\lambda}(x,y)A(x,y)\lambda(x,y)$$

integrate the compact space $\Rightarrow$ 4D theory

$$L_4 = Y\int d^4x \overline{\chi}(x)\varphi(x)\chi(x)$$

$$Y = g\int d^6y \overline{\psi}(y)\phi(y)\psi(y)$$

Coupling is obtained by the overlap integral of wavefunctions

Chiral theory

When we start with extra dimensional field theories,

how to realize chiral theories is one of important issues from the viewpoint of particle physics.

$$\boxed{i\gamma^m D_m \psi = 0}$$

Zero-modes between chiral and anti-chiral fields are different from each other on certain backgrounds,

e.g. CY, toroidal orbifold, warped orbifold, magnetized extra dimension, etc.

Magnetic flux

$$i\gamma^m D_m \psi = 0$$

The limited number of solutions with non-trivial backgrounds are known.

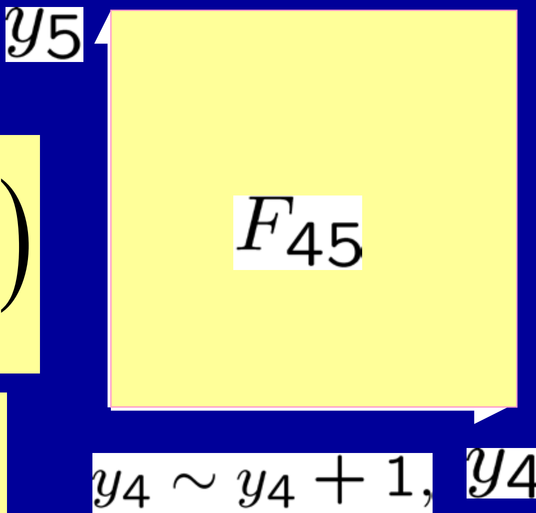
Generic CY is difficult.

Toroidal/Wapred orbifolds are well-known.

Background with magnetic flux is one of interesting backgrounds.

量子力学の復習:磁場中の粒子(Landau)　$U(1)$

$$F_{45}=2\pi M,\ A_4=0,\ A_5=2\pi M y_4$$



$$H=\frac{1}{2m}\left(P_4^2+(P_5-2\pi M y_4)^2\right)$$

$$[H,P_5]=0 \quad \Downarrow \quad P_5=2\pi k$$

$$y_4 \sim y_4+1,\quad y_5 \sim y_5+1$$

$$H=\frac{1}{2m}\left(P_4^2+4\pi^2M^2(y_4-k/M)^2\right)$$

座標がk/Mずれた調和振動子　　M=整数

M個の基底状態　　k=0,1,2,…………,(M-1)

Higher Dimensional SYM theory with flux

Cremades, Ibanez, Marchesano, '04

4D Effective theory <= dimensional reduction

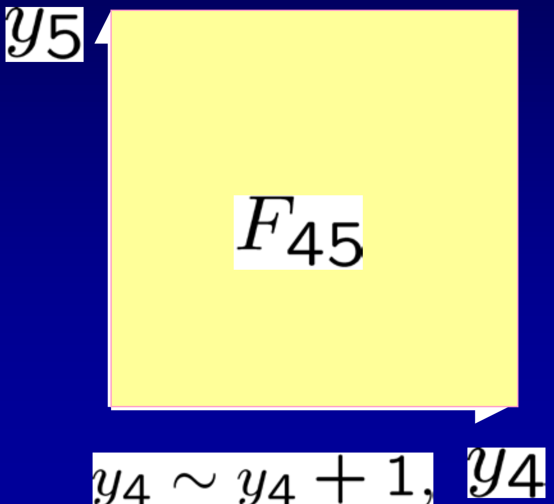
$$\mathcal{L}_{SYM} = -\frac{1}{4g^2} Tr\{F^{MN}F_{MN}\} + \frac{i}{2g^2} Tr\{\bar{\lambda}\Gamma^M D_M \lambda\}$$

$$\begin{aligned} \lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\ A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m) \end{aligned} \Rightarrow \begin{aligned} i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\ \Delta_6 \phi_{n,M}(y) &= M^2_{n,M} \phi_{n,M} \end{aligned}$$

The wave functions $\longrightarrow$ eigenstates of corresponding internal Dirac/Laplace operator.

Higher Dimensional SYM theory with flux $U(1)$

Abelian gauge field on magnetized torus T^2



$$y_4 \sim y_4 + 1, \quad y_5 \sim y_5 + 1$$

Constant magnetic flux $F_{45} = b,$

gauge fields of background $\begin{cases} A_4 = 0, \\ A_5 = b y_4 \end{cases}$

$$\Downarrow \qquad \frac{b}{2\pi} = M \in Z$$

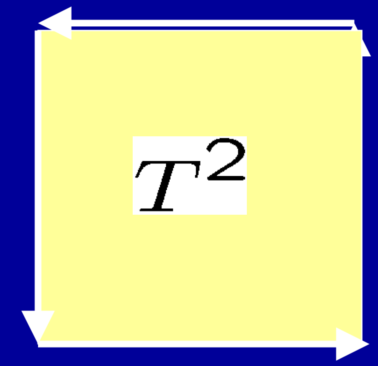
The boundary conditions on torus (transformation under torus translations)

$$\begin{cases} A_m(y_4 + 1, y_5) = A_m(y_4, y_5) + \partial_m \chi_4, & \chi_4 = b y_5, \\ A_m(y_4, y_5 + 1) = A_m(y_4, y_5) + \partial_m \chi_5, & \chi_5 = 0, \end{cases}$$

Higher Dimensional SYM theory with flux $U(1)$

We now consider a complex field $\psi(y_4, y_5)$ with charge Q (+/-1)

$$\begin{cases} \psi(y_4+1, y_5) = e^{iQ\chi_4}\psi(y_4, y_5) = e^{iQby_5}\psi(y_4, y_5), \\ \psi(y_4, y_5+1) = e^{iQ\chi_5}\psi(y_4, y_5) = \psi(y_4, y_5), \end{cases}$$



Consistency of such transformations under a contractible loop in torus which implies Dirac’s quantization conditions.

$$\boxed{\frac{b}{2\pi} = M \in Z}$$

Dirac equation on 2D torus

ψ is the two component spinor.

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

U(1) charge Q=1

$$\begin{cases} \left[\bar{\partial} + 2\pi M y_4\right]\psi_+(y) = 0 \\ \left[\partial - 2\pi M y_4\right]\psi_-(y) = 0 \end{cases}$$

$$\partial = \partial_4 + i\partial_5, \qquad \bar{\partial} = \partial_4 - i\partial_5$$

with twisted boundary conditions (Q=1)

$$\begin{aligned} \psi(y_4+1, y_5) &= e^{2\pi i M y_5}\psi(y_4, y_5), \\ \psi(y_4, y_5+1) &= \psi(y_4, y_5), \end{aligned}$$

Dirac equation and chiral fermion

|M| independent zero mode solutions in Dirac equation.

$$\Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \cdot \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)$$

$$(j = 0, 1, \cdots, |M| - 1)$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) \equiv \sum_n e^{\pi i(n+a)^2\tau} e^{2\pi(a+n)(\nu+b)}$$

(Theta function)

Properties of theta functions

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m, \tau) = e^{2\pi ima} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m\tau, \tau) = e^{-\pi m^2\tau - 2\pi im(\nu+b)} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

chiral fermion

$M \gtrless 0 \Rightarrow$

$\psi_{+/-}$:Normalizable mode

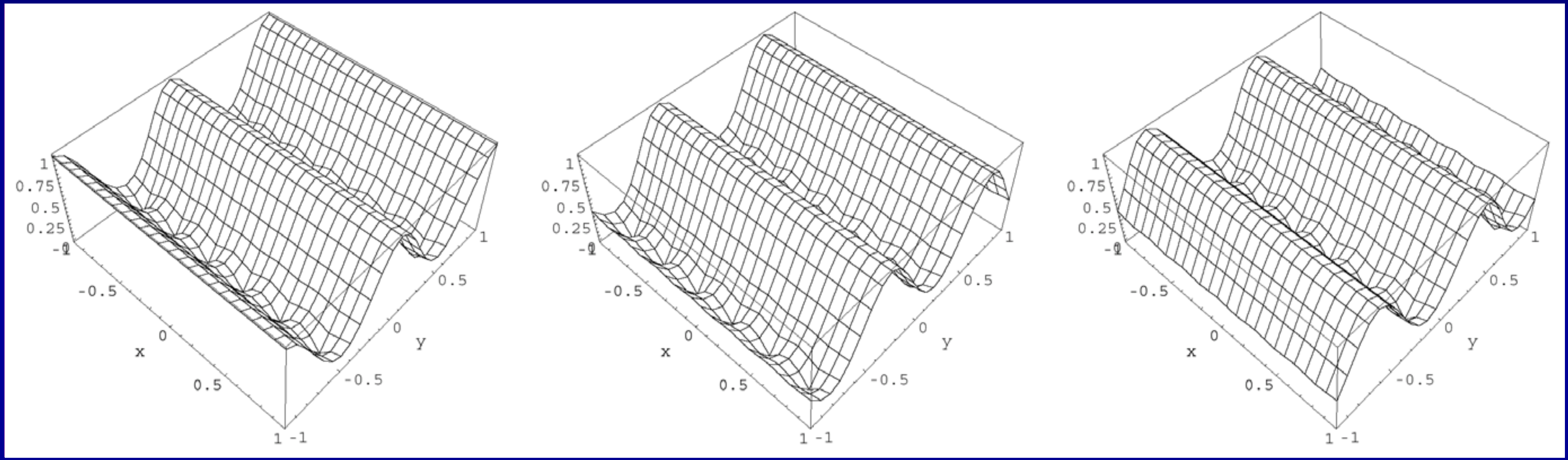
$\psi_{-/+}$:Non-normalizable mode

By introducing magnetic flux, we can obtain chiral theory.

Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Fermions in bifundamentals $(N = N_a + N_b)$

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a\times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b\times N_b} \end{pmatrix}.$$

Breaking the gauge group $U(N) \to U(N_a) \times U(N_b)$

(Abelian flux case $M_a, M_b \in Z$

The gaugino fields $\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$

$\Rightarrow$ λ^{aa} and λ^{bb} **gaugino of unbroken gauge** Adj N_a, Adj N_b.

λ^{ab} and λ^{ba} **bi-fundamental matter fields** $(N_a, \bar{N}_b), (\bar{N}_a, N_b)$.

If both appear in 4D, that is non-chiral theory.

Zero-modes Dirac equations

$$\begin{pmatrix} \bar{\partial}\psi_+^{aa} & \left[\bar{\partial} + 2\pi(M_a - M_b)y_4\right]\psi_+^{ab} \\ \left[\bar{\partial} + 2\pi(M_b - M_a)y_4\right]\psi_+^{ba} & \bar{\partial}\psi_+^{bb} \end{pmatrix} = 0.$$

$$\begin{pmatrix} \partial\psi_-^{aa} & \left[\partial - 2\pi(M_a - M_b)y_4\right]\psi_-^{ab} \\ \left[\partial - 2\pi(M_b - M_a)y_4\right]\psi_-^{ba} & \partial\psi_-^{bb} \end{pmatrix} = 0.$$

No effect due to magnetic flux for adjoint matter fields, λ^{aa} and λ^{bb}

Total number of zero-modes of λ^{ab} $\Rightarrow$ $I_{ab} = |M_a - M_b|.$

$M_a - M_b > 0 \Rightarrow$

ψ_+^{ab}, ψ_-^{ba} :Normalizable mode

ψ_-^{ab}, ψ_+^{ba} :Non-Normalizable mode

4D chiral theory

10D spinor

light-cone 8s $(\pm,\pm\ \pm\ \pm)$

even number of minus signs

1st $\Rightarrow$ 4D, the other $\Rightarrow$ 6D space

If all of λ^{ab} and λ^{ba} $(N_a, \bar{N}_b), (\bar{N}_a, N_b).$ appear in 4D theory, that is non-chiral theory.

If $M_a - M_b > 0 \Rightarrow$ for all torus,

only

$$\lambda^{ab}\left(N_a, \overline{N}_b\right)(+,+\ +\ +)$$

appear for 4D helicity fixed.

$\Rightarrow$ 4D chiral theory

Wilson lines

Cremades, Ibanez, Marchesano, '04,

Abe, Choi, T.K. Ohki, '09

torus without magnetic flux

constant Ai → mass shift

every charged modes massive

magnetic flux

$$\left[\bar{\partial} + 2\pi(My + a)\right]\psi_+ = 0$$

$$\left[\partial - 2\pi(My + a)\right]\psi_- = 0$$

the number of zero-modes is the same.

the profile: f(y) → f(y +a/M)

with proper b.c.

U(1)a*U(1)b theory (or U(2) theory)

magnetic flux, Fa=2πM, Fb=0

Wilson line, Aa=0, Ab=C

matter fermions with U(1) charges, (Qa,Qb)

chiral spectrum,

for Qa=0, massive due to nonvanishing WL

when MQa >0, the number of zero-modes is MQa.

zero-mode profile is shifted depending on Qb,

$$f(z) \Rightarrow f(z + CQ_b/(MQ_a))$$

Pati-Salam model

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & m_2 \mathrm{I}_{N_2} & \\ 0 & & m_3 \mathrm{I}_{N_3} \end{pmatrix}$$

$$N_1 = 4,\ N_2 = 2,\ N_3 = 2$$

Pati-Salam group

$$U(4) \times U(2)_L \times U(2)_R$$

$$(m_1 - m_2) = (m_3 - m_1) = 3 \text{ for the first } T^2$$

$$(m_1 - m_2) = (m_3 - m_1) = 1 \text{ for the other tori}$$

$$\text{three families of } (4,2,1) + (\bar{4},1,2)$$

WLs along a U(1) in U(4) and a U(1) in U(2)R

=> Standard gauge group up to U(1) factors

$$U(3)_C \times U(2)_L \times U(1)^3$$

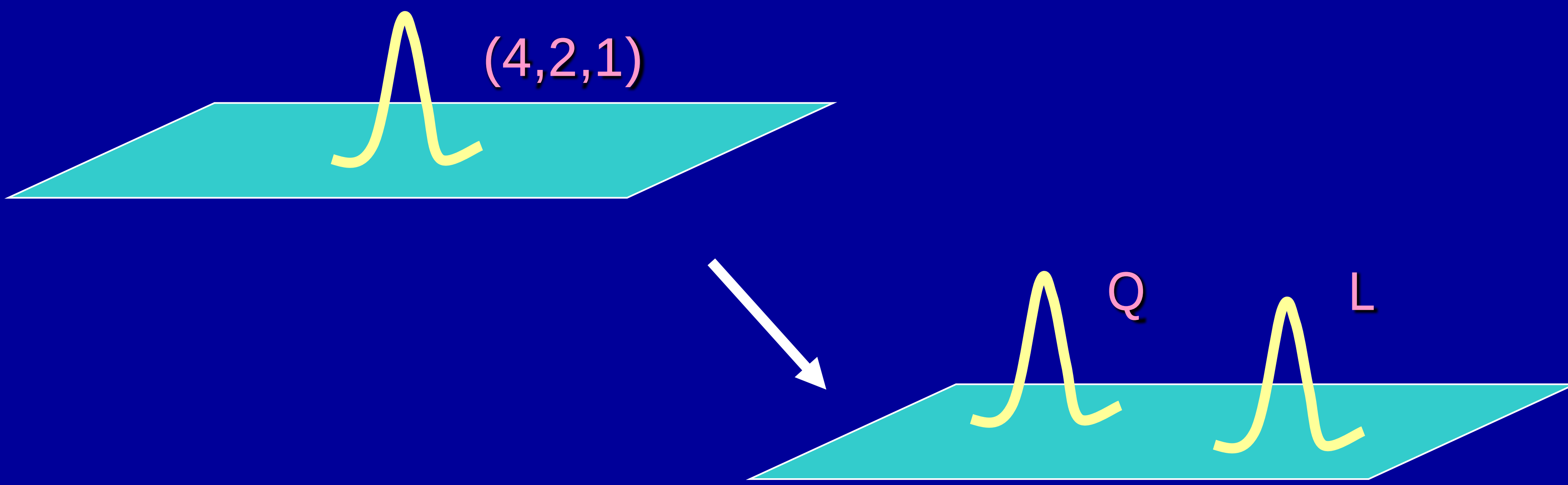
U(1)Y is a linear combination.

PS => SM

$$(4,2,1)+(\overline{4},1,2)$$

Zero modes corresponding to three families of matter fields remain after introducing WLs, but their profiles split

$$(4,2,1)=(3,2,1)+(1,2,1)$$
$$(\overline{4},1,2)=(\overline{3},1,1)+(\overline{3},1,1)+(1,1,1)+(1,1,1)$$



3. SO(32) heterotic string theory

Heterotic string theory

E8xE8 vs SO(32)

→ E8xE8 or SO(32) 10D N=1 SYM

E8 > E7 > E6 > SO(10) > SU(5) > SU(3)xSU(2)xU(1)

E8 adj. includes SO(10) 16, which is one generation
of quarks and leptons
including right-handed neutrinos

SO(32) adj. does not include SO(10) 16.

People preferd E8xE8 heterotic string theory
to SO(32) theory in order to derive the SM.

SO(32) adjoint representation

16 U(1)'s in SO(32)

We introduce magnetic fluxes along 13 U(1)' among 16 U(1)'s.

SO(32) breaks to SU(3)xSU(2)xU(1) with 12 U(1)'s.

$$
\begin{aligned}
SO(32)\,\mathrm{adj.} &= 2\mathrm{x}(3,2) + 2\mathrm{x}(\overline{3},2) \\
&\quad + \mathrm{several}(3,1) + \mathrm{several}(1,2) + \ldots\ldots \\
&\quad + (\mathrm{lots\ of\ singlets})
\end{aligned}
$$

Magnetic fluxes determines the number of chiral zero-modes

We search magnetic fluxes such that three chiral generations of quarks and leptons appear.

10D effective field theory

10D action at tree level

$$\frac{1}{2\kappa_{10}^2}\int e^{-2\phi}\left[R + 4d\phi \wedge *d\phi - \frac{1}{2}H \wedge *H\right]$$
$$-\frac{1}{4g_{10}^2}\int e^{-2\phi} tr(F \wedge *F)$$

$$H = dB - \frac{\alpha'}{4}(\omega_{YM} - \omega)$$

gauge and gravitational Chern-Simons terms

$$d\omega_{YM} = tr(F \wedge F), \qquad d\omega = tr(R \wedge R)$$

10D effective field theory

Green-Schwarz term

$$\frac{1}{24(2\pi)^5\alpha'}\int B\wedge X_8$$

$$X_8=\frac{1}{24}TrF^4-\frac{1}{7200}(TrF^2)^2-\frac{1}{240}(TrF^2)(trR^2)+\frac{1}{8}trR^4+\frac{1}{32}(trR^2)^2$$

For some U(1) factors,

there appear BF couplings in 4D action.

That makes U(1) gauge bosons massive

Massive U(1) gauge bosons

Green-Schwarz mechanism

$$(dB)^2 - 2cBF$$

$$\Longrightarrow$$

$$(dB - cA)^2 - c^2 A^2$$

U(1)Y should not couple to B.

Bii (i=1,2,3) on the i-th torus.

B for 4D

Totally, there are four conditions on magnetic fluxes.

K-theory condition

SO(32) heterotic string

S-dual type I

type IIB D9-brane system

D5-brane, → NS five-brane in hetero.

Introduction of all possible probe D-brane

→ constraints due to discrete K-theory charge, Z2

such as Witten anomaly

→ constraints on magnetic fluxes

NS five brane

10D action

$$-\frac{1}{4\kappa_{10}^2}\int e^{2\phi} dB^{(6)} \wedge * dB^{(6)}$$
$$+\frac{\alpha'}{8\kappa_{10}^2}\int e^{-2\phi} B^{(6)} \wedge \left(trF^2 - trR^2 - 4(2\pi)^2 \sum N_a \delta(\Gamma_a) \right)$$

$$* dB = e^{2\phi} dB^{(6)}$$

tadpole candellation condition

$$\int \left(trF^2 - 4(2\pi)^2 \sum N_a \delta(\Gamma_a) \right) = 0$$

If only magnetic fluxes themselves satisfy this condition, we don't need five-branes, otherwise we need five-branes.

Model search

We have found many models

the gauge symmetry including SU(3)xSU(2)xU(1)Y

exactly three chiral generations of quarks and leptons

and vector-like matter including higgs (higgsino)

and lots of singlets.

In many models, the gauge symmetry is enhanced

to SU(4)xSU(2)xXU(2) with non-Abelian hidden groups

Wilson lines break SU(4)xSU(2)xSU(2)

to SU(3)xSU(2)xU(1)Y.

Model

Magnetic fluxes along 13U(1) directions

$$2m_1 = (1,-3,-1), \quad 2m_2 = (7,-1,1), \quad m_3 = (0,0,0)$$
$$2m_4 = -2m_9 = (-5,-1,5)$$
$$2m_{5,6,7,8} = -2m_{10,11,12,13,14} = (3,1,1)$$

with Wilson line breaking

the gauge symmetry SU(3)xSU(2)xU(1)y

exactly three chiral generations of quarks and leptons

vector-like matter and singlets

There are many models with such aspects.

Flavor structure

in some models, the number of generations is obtained
by magnetic fluxes
and those quarks and /or leptons have the same
extra U(1) charges.

In other models, three families of quarks and/or leptons
have different extra U(1) charges.

For example, there are models with SU(3) flavor symmetries
for right-handed quarks and leptons
Wilson line breaking and another breaking

Next

We have constructed the models with realistic
massless spectrum.
SU(3)x SU(2)xU(1)Y gauge symmetry
exactly three chiral generations of
quarks and leptons,
and vector-like matter and singlets.

What is next ?
Are these models realistic quantitatively ?

Gauge couplings, quarks/lepton masses and mixing angles,
Higgs potential,,,,,,,,,

4. Gauge coupling unification

10D action at tree level

$$-\frac{1}{4g_{10}^2}\int e^{-2\phi}tr(F\wedge *F)$$

4D action

$$-\frac{1}{4g_4^2}F_{\mu\nu}F^{\mu\nu}$$

$$\frac{1}{g_4^2}\propto e^{-2\phi}V_6$$

$$\frac{1}{g_4^2}=\mathrm{Re}(S),$$

$$\text{gauge kinetic function}\quad f=S$$

Gauge couplings are unified at the string scale.

Universal axion

10D action

$$\frac{\alpha'}{8\kappa_{10}^2}\int e^{-2\phi}B^{(6)}\wedge\left(trF^2 - trR^2 - 4(2\pi)^2\sum N_a\delta(\Gamma_a)\right)$$

$$B^{(6)} = b\times(\text{6D volume form})+\ldots.$$

4D action

$$bF\tilde{F}$$

$$\frac{1}{g_4^2} = \mathrm{Re}(S),$$

$$\text{gauge kinetic function}\quad f = S$$

$$\mathrm{Im}(S)\propto b$$

Gauge couplings

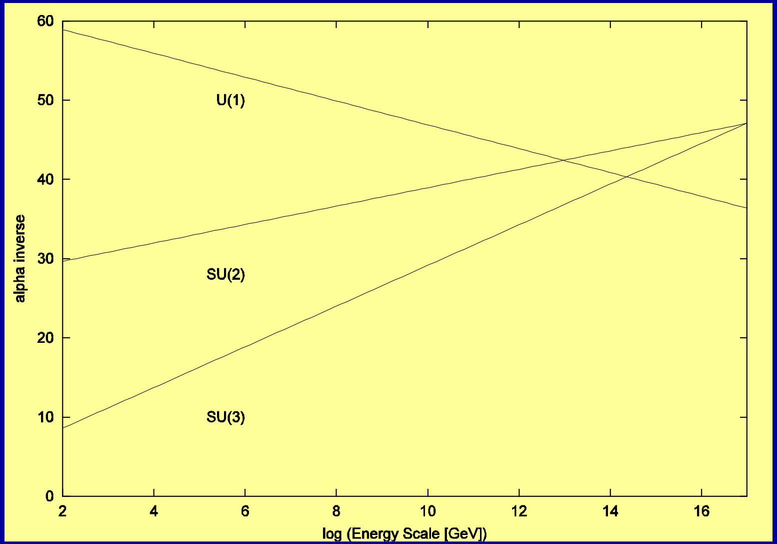
experimental values

RG flow ⇒

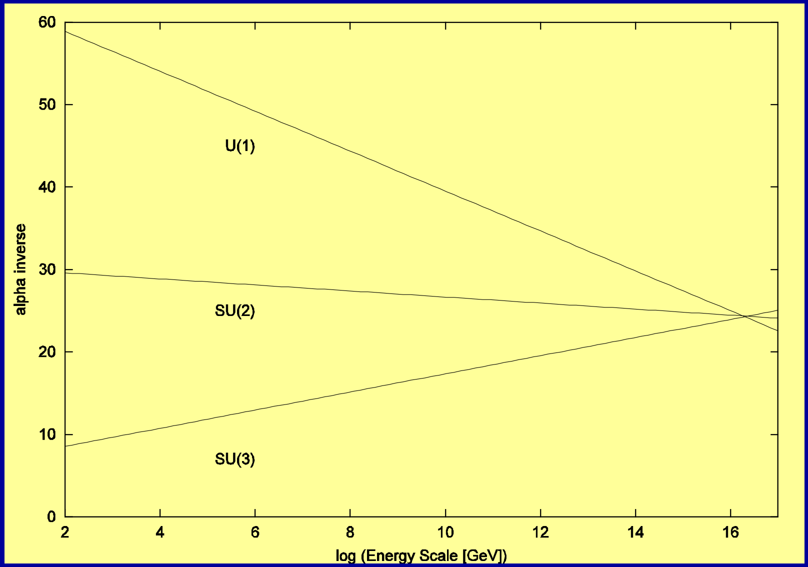
They approach each other

and become similar values

at high energy



U(1)Y GUT normalization



Gauge coupling unification

10D effective field theory

Green-Schwarz term

$$\frac{1}{24(2\pi)^5\alpha'}\int B\wedge X_8$$

$$X_8=\frac{1}{24}TrF^4-\frac{1}{7200}(TrF^2)^2$$
$$-\frac{1}{240}(TrF^2)(trR^2)+\frac{1}{8}trR^4+\frac{1}{32}(trR^2)^2$$

Because of magnetic fluxes,

Bii couples to SU(3) and SU(2) non-universally.

Axions

Green-Schwarz term

$$\frac{1}{24(2\pi)^5\alpha'}\int B\wedge X_8$$

$$X_8=\frac{1}{24}TrF^4-\frac{1}{7200}(TrF^2)^2 -\frac{1}{240}(TrF^2)(trR^2)+\frac{1}{8}trR^4+\frac{1}{32}(trR^2)^2$$

$$B=\alpha' b_i\omega_i+.....$$

2D volume form corresponding to i-th 2D torus

bi couples to SU(3) and SU(2) non-universally.

Non-universal gauge kinetic functions

Moduli dependence

$$T_k = t_k + i b_k$$

$$f_{SU(3)} = S + \beta_k^3 T_k$$

$$f_{SU(2)} = S + \beta_k^2 T_k$$

$$\beta_k^3 = \frac{1}{2\pi} m_2^i m_2^j, \qquad \beta_k^2 = \frac{1}{2\pi} m_1^i m_1^j,$$

$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{N} U(1)_c \right)$$

$$f_Y = \left(\frac{1}{6} + \frac{N-3}{2} \right) S$$

Gauge coupling unification

In a certain model with specific values of moduli,
we can fit the gauge couplings to experimental values.

SUSY

Non-vanishing magnetic fluxes induce D-term
(moduli-dependent FI -term)

$$D_a = \sum_{i=1}^{3} \frac{m_a^i}{A_i}$$

MSSM: We assume that these are vanishing.

SM : We assume that these are not vanishing.

Moduli values

In a certain model with specific values of moduli,
we can fit the gauge couplings to experimental values.

For fixed magnetic fluxes,
there are three conditions (gauge couplings)
among four parameters, S, T1, T2, T3,
e.g. for MSSM

$$S = 2.47$$
$$T_2 = 0.86 T_1 + 0.21$$
$$T_3 = 0.71 T_1 + 0.81$$

$$(2m_1^1, 2m_1^2, 2m_1^3) = (1, -3, -1)$$
$$(2m_2^1, 2m_2^2, 2m_2^3) = (7, -1, 1)$$

All of the moduli values as well as V6
can be of O(1) .

Summary

We have studied SO(32) heterotic string theory with magnetic fluxes.

We can realize

the SU(3)xSU(2)xU(1)Y gauge symmetry,

exactly three chiral generations of quarks and leptons

vector-like matter and singlets.

Gauge couplings have non-universal corrections.

One can fit them to experimental values

by using free parameters, moduli.

Further studies

Massless spectrum is realistic.
Gauge couplings can be realistic.

What is next ?
Derivation of realistic values of quark/lepton masses and mixing angles.

Moduli stabilization

SUSY breaking and SUSY spectrum