Coleman-Weinberg inflation

Iso, KK, Shimada (2014)

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THE GRADUATE UNIVERSITY FOR ADVANCED STUDIES [SOKENDAI]

Abstract (1/3)

- Coleman-Weinberg conformal model is attractive to induce the EW phase transition. We need a B-L scalar to overcome the top Yukawa's negative contribution to $\beta_{\lambda H}$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

$$\beta(\phi) = \frac{\partial \lambda(\phi)}{\partial \ln \phi} > 0$$

$$V_{\rm cl} = \lambda_H |H|^4 + \lambda_\Phi |\Phi|^4 + \lambda_{mix} |H|^2 |\Phi|^2$$

$\lambda_{mix} \sim -10^{-5}$

λ mix is negative at $\langle \phi \rangle = M$

We assume $\lambda_H \gg \lambda_\Phi$ and $|\lambda_{mix}|$

$\lambda_H(M_{UV}) = \lambda_{mix}(M_{UV}) = 0$ at the UV scale, e.g. $M_{UV} = M_{\rm Pl}$

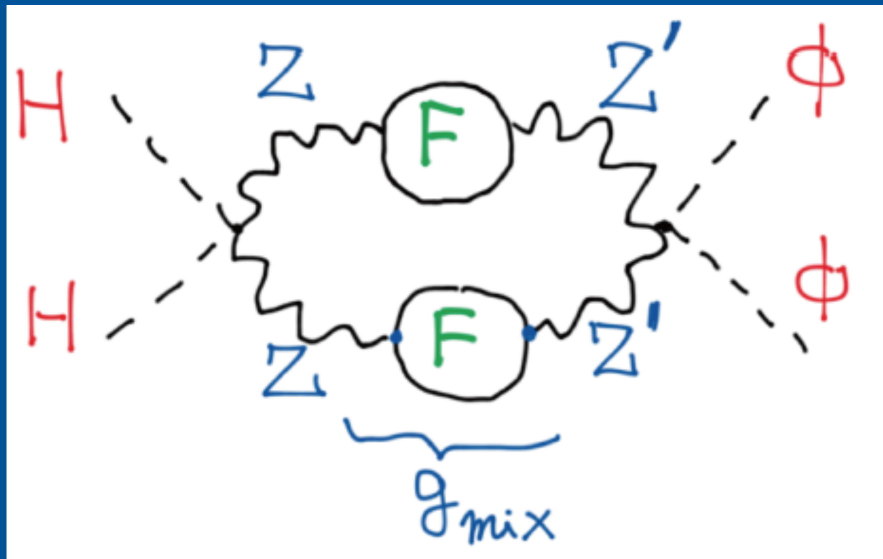
Abstract (2/3)

- Mixing term (and even λ_Φ in flatland) is derived only by radiative corrections

$$\lambda_{mix} \approx -6 \cdot 10^3 \times \alpha_{B-L}^2 \alpha_Y^2$$

$$\alpha_{B-L} \sim 10^{-3}$$

$$\lambda_{mix} \sim -10^{-5}$$

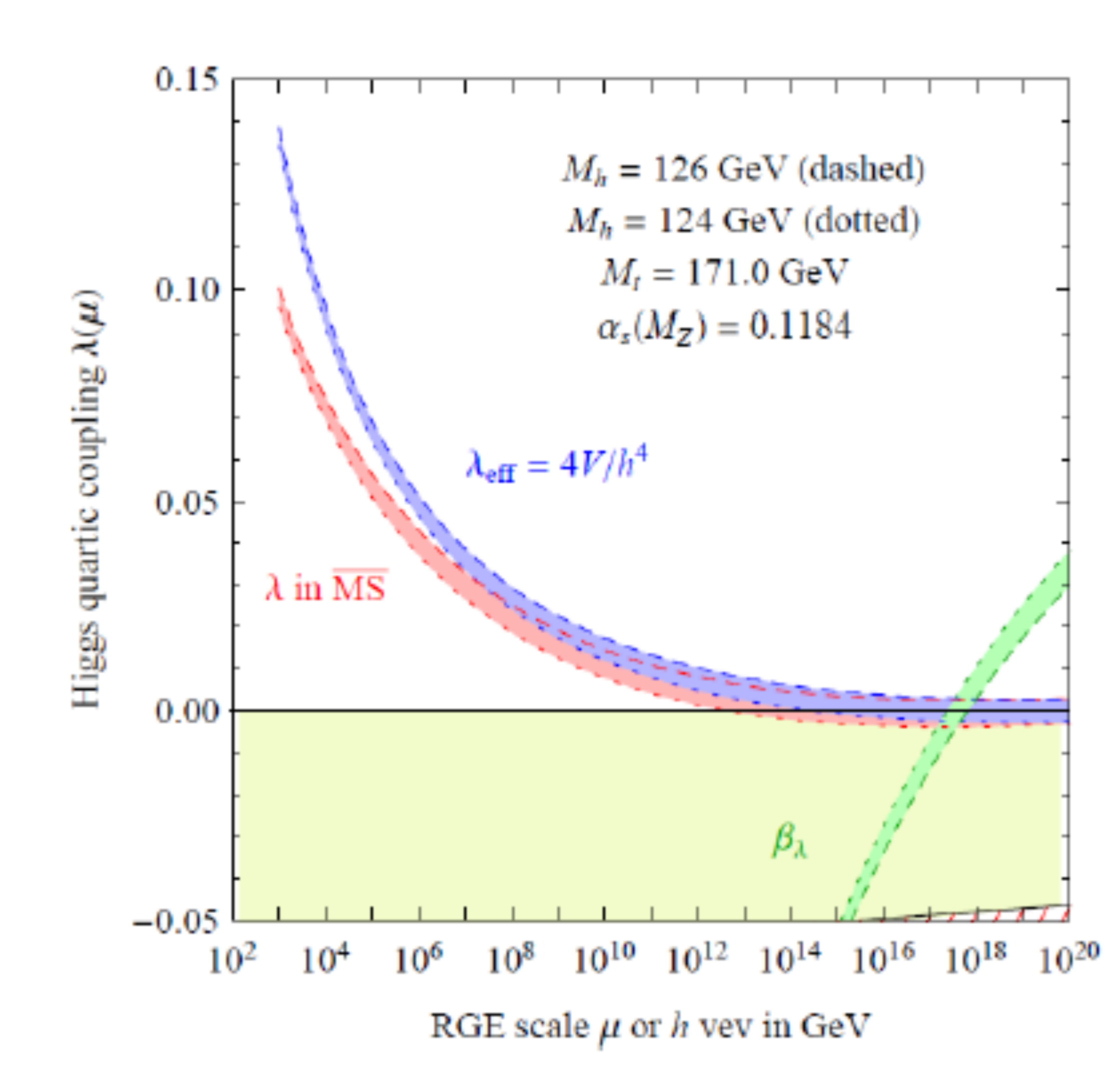


A figure by Iso-san's

Abstract (3/3)

- No other scales than Planck mass (no hierarchy problem)
- We have flat potentials for Higgs and B-L scalar ϕ up to the Planck scale,
- or zero at Planck scale (flatland scenarios)

Flat at Planck scale?



$$\beta(h) = \frac{\partial \lambda(h)}{\partial \ln h}$$

In standard model (not included U(1)B-L scalar)

Coleman-Weinberg potential

$$\beta(\phi) = \frac{\partial \lambda(\phi)}{\partial \ln \phi}$$

- One-loop potential

$$V(\phi) = \frac{A}{4}\phi^4 \left(\ln \frac{\phi^2}{M^2} - \frac{1}{2} \right) + V_0, \quad V_0 = \frac{AM^4}{8}$$

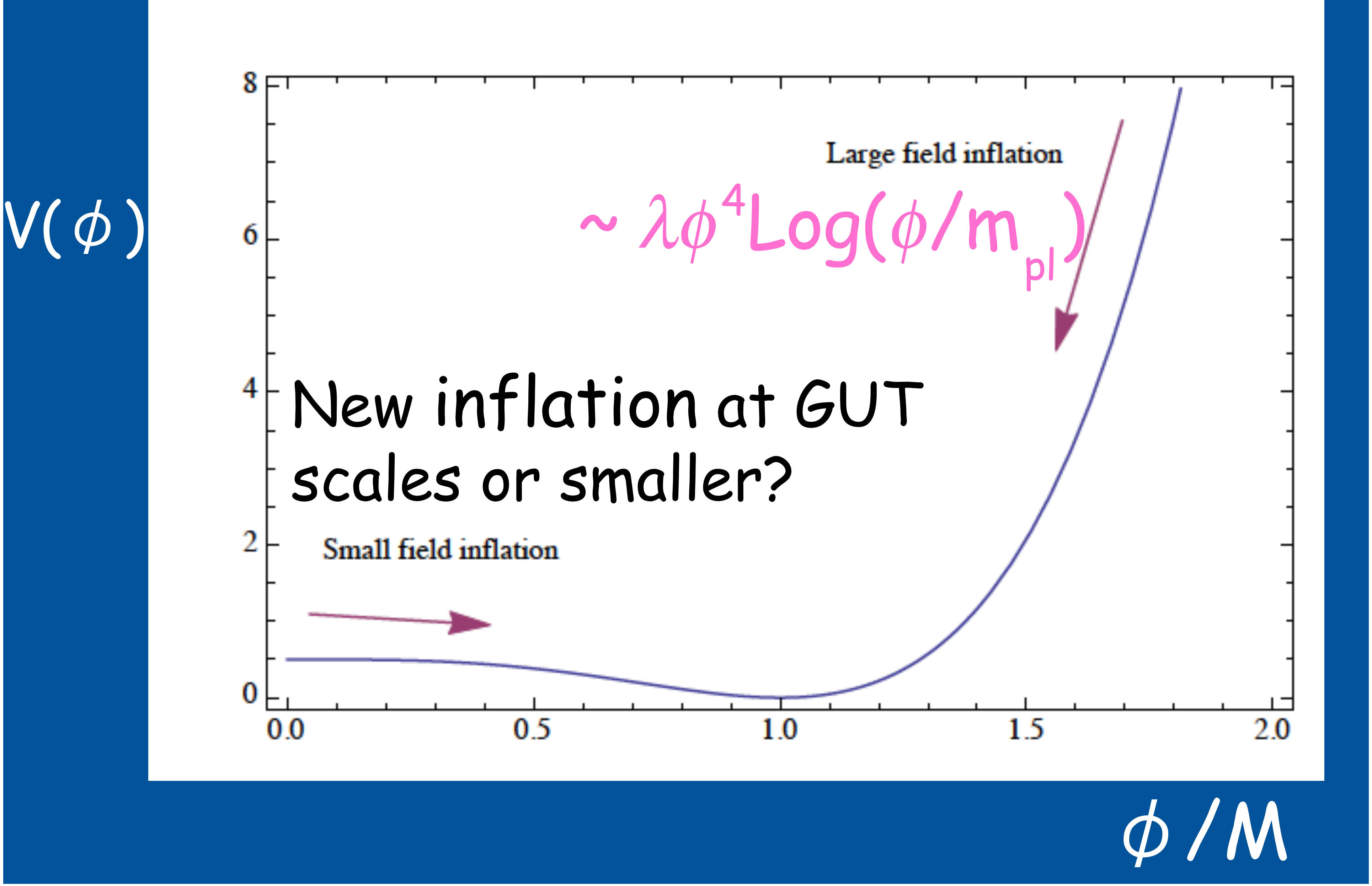
Minimum: $\phi = M$ $\quad 6\lambda = V^{(4)}(M) = 22A$ and $\beta_\phi = 2A$

- Derivatives

$$V' = A\phi^3 \ln \frac{\phi^2}{M^2}$$

$$V'' = A\phi^2 \left(2 + 3 \ln \frac{\phi^2}{M^2} \right)$$

Potential $V(\phi)$



Small-field Coleman-Weinberg Conformal models

Iso, KK, Shimada (2014)

Problems in Coleman-Weinberg inflation

- Perturbation ($\propto$ V/ε) with a potential $V(\phi)$ at the GUT scale is too large for fixed e-folding number N ~ 50 -60, due to the smallness of ε
- Even if we reduces the energy scale of $V(\phi)$, e.g. down to TeV scale, the amplitude of perturbation is well fitted, but the spectral index is too small, $n_s \ll 0.94$ (A new severe problem)

Slowroll parameters

- 1st slowroll parameter

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \approx 32\left(\frac{M_{\rm Pl}}{M}\right)^2\left(\frac{\phi}{M}\right)^6\left(\ln\frac{\phi^2}{M^2}\right)^2$$

- 2nd slowroll parameter

$$\eta = M_{\rm Pl}^2\left(\frac{V''}{V}\right) \approx 24\left(\frac{M_{\rm Pl}}{M}\right)^2\left(\frac{\phi}{M}\right)^2 \ln\frac{\phi^2}{M^2}$$

$$\phi \ll M \sim 10^7\,\mathrm{GeV} - 10^{11}\,\mathrm{GeV}$$

$$\rightarrow \varepsilon \ll |\eta|$$

Field values

$$\phi^2/M^2 \approx (|\eta|/24\ln(24M_{Pl}^2/|\eta|M^2))(M/M_{Pl})^2$$
$$\approx 10^{-3}|\eta|(M/M_{Pl})^2 \ll 1.$$

For $\phi \ll M$, and $\eta \sim -0.02$

$$\epsilon = \frac{|\eta|^3}{432\ \ln(24M_{Pl}^2/|\eta|M^2)}\left(\frac{M}{M_{Pl}}\right)^4 \ll 1$$

Field values

- Normalization of the perturbation at pivot

$$\Delta_R^2 \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^4 \epsilon} = \frac{9A\ln(24M_{Pl}^2/|\eta|M^2)}{4\pi^2|\eta|^3}$$

$$= 2.215 \times 10^{-9}$$

$$k_0 = k_{\rm CMB} = 0.05\ {\rm Mpc}^{-1}$$

- Parameters

$$A \sim 10^{-15}$$

$$V_0^{1/4} \sim 10^{-4} M$$

100 TeV for $M = 10^9$GeV

E-folding number

- E-folding number

$$N = \frac{1}{M_{\rm Pl}^2}\int_{\phi_{end}}^{\phi} \frac{V}{V'}d\phi \approx \frac{3}{2}\left(\frac{1}{|\eta|} - \frac{1}{|\eta_{end}|}\right)$$

$$N = 3/(1-n_s) - 3/2 = 73.5$$

For $1\text{-}n_s$=0.04

- Required value

$$N > N_{CMB} = 61 + \frac{2}{3}\ln\left(\frac{V_0^{1/4}}{10^{16}GeV}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{16}GeV}\right)$$

$$\sim 30 + \frac{2}{3}\ln\left(\frac{V_0^{1/4}}{10^{3}GeV}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{3}GeV}\right)$$

Fermion condensate

Iso, KK, Shimada (2014)

with $H_{\rm inf} \ll 100\,\mathrm{MeV}$

- We need a negative linear term

$$V_{\rm linear} \sim -C\phi \sim -C_0 h$$

- Quark condensates and mixing between Higgs and ϕ $\lambda_{\rm mix}\varphi^2 h^2$

$$V_{\rm linear} \sim -y < \bar{q}q > h$$

$$C_0 \sim y\langle \bar{q}q\rangle$$

y: Yukawa coupling

$$C = \sqrt{|\lambda_{mix}|/2\lambda_h} C_0 = (246/M[\mathrm{GeV}])C_0$$

E-folding number

$$N = \frac{1}{M_{Pl}^2} \int_{\phi}^{\phi_{end}} \frac{V_0 d\phi}{A\phi^3 \ln(M^2/\phi^2) + C}$$

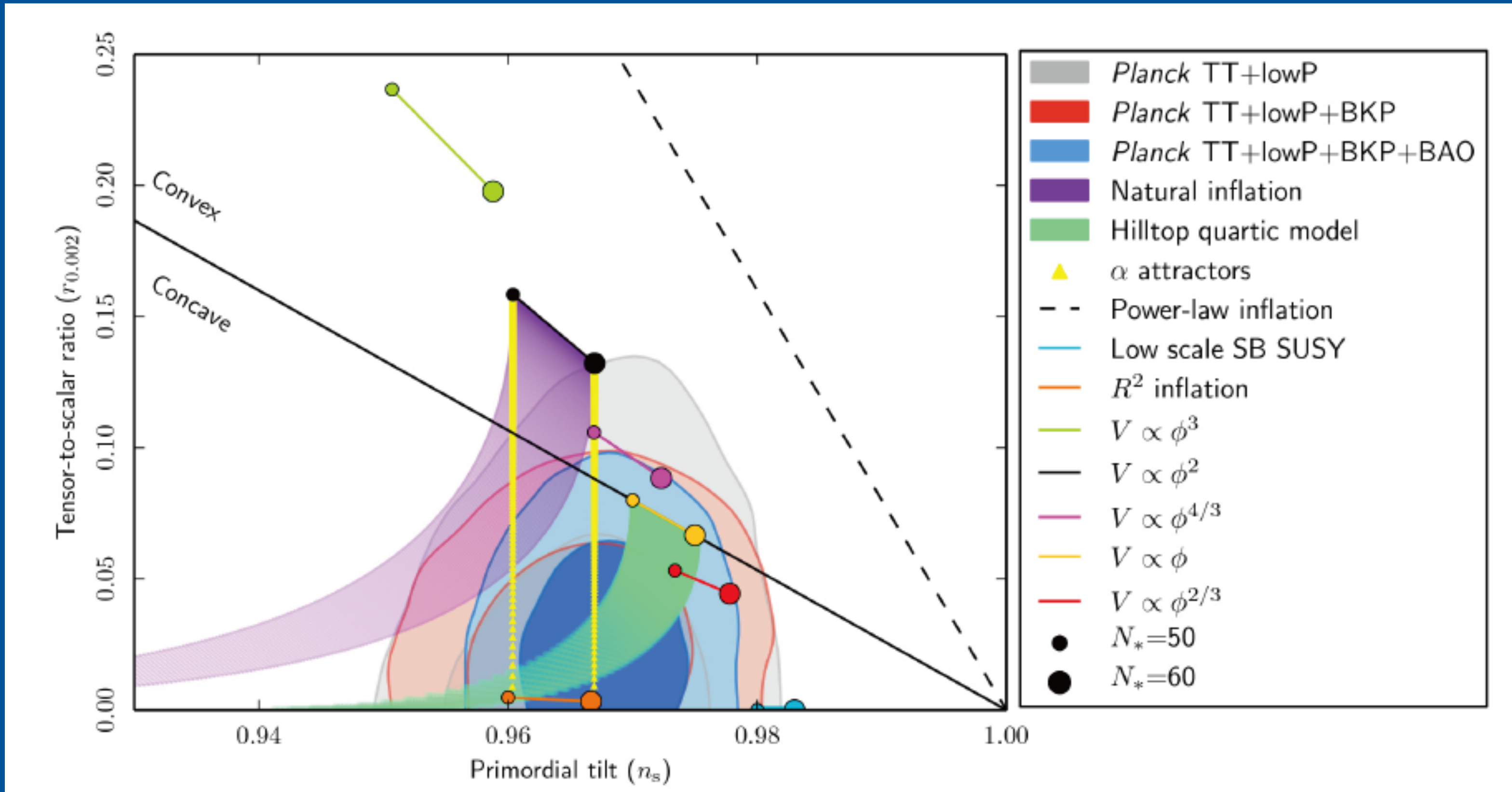
$$C \equiv A\tilde{C}M^3(M/M_{\rm Pl})^3$$

the denominator of N balance for $C \sim 10^{-6}$ and $\phi_{\rm CMB}$, or for $\widetilde{C} \sim 10^{-3}$ and ϕ_{end}.

$$y \sim 10^{-6}, \quad \langle \bar{q}q \rangle \sim (100\,MeV)^3, \quad \tilde{C} \sim 10^{-5} \leftrightarrow M = 10^8\,GeV$$

Tensor to scalar ratio by Planck and BICEP/Keck + BAO

Planck 2015 results. XX. Constraints on inflation



Strong point: monomial $1/2m^2\varphi^2$ was excluded at two sigma

Running of Spectral Index

$$\alpha \equiv \frac{dn_s}{d\ln k} = -24\varepsilon^2 + 16\varepsilon\eta - 2\xi^{(2)}$$

$$\sim -O(10) \times O(10^{-2})^2$$

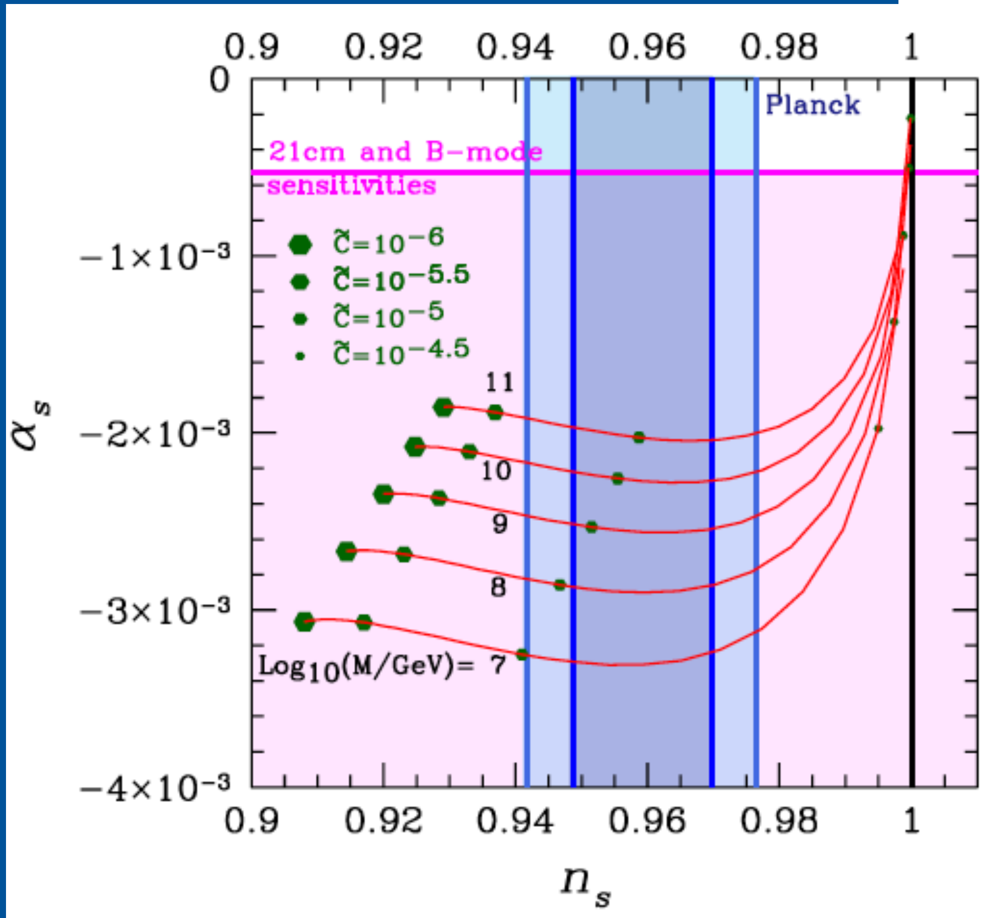
$$\sim -O(10^{-3})$$

$$\xi^{(2)} \equiv \frac{V'''V'}{V^2} M_{\mathrm{P}}^4$$

Running

$$\alpha_s \approx -2\xi^{(2)} \text{ with } \xi^{(2)} \equiv V'V'''M_{\rm Pl}^4/V^2$$

$$C \equiv A\tilde{C}M^3(M/M_{\rm Pl})^3$$



Iso, KK, Shimada (2014)

What about smaller scales?

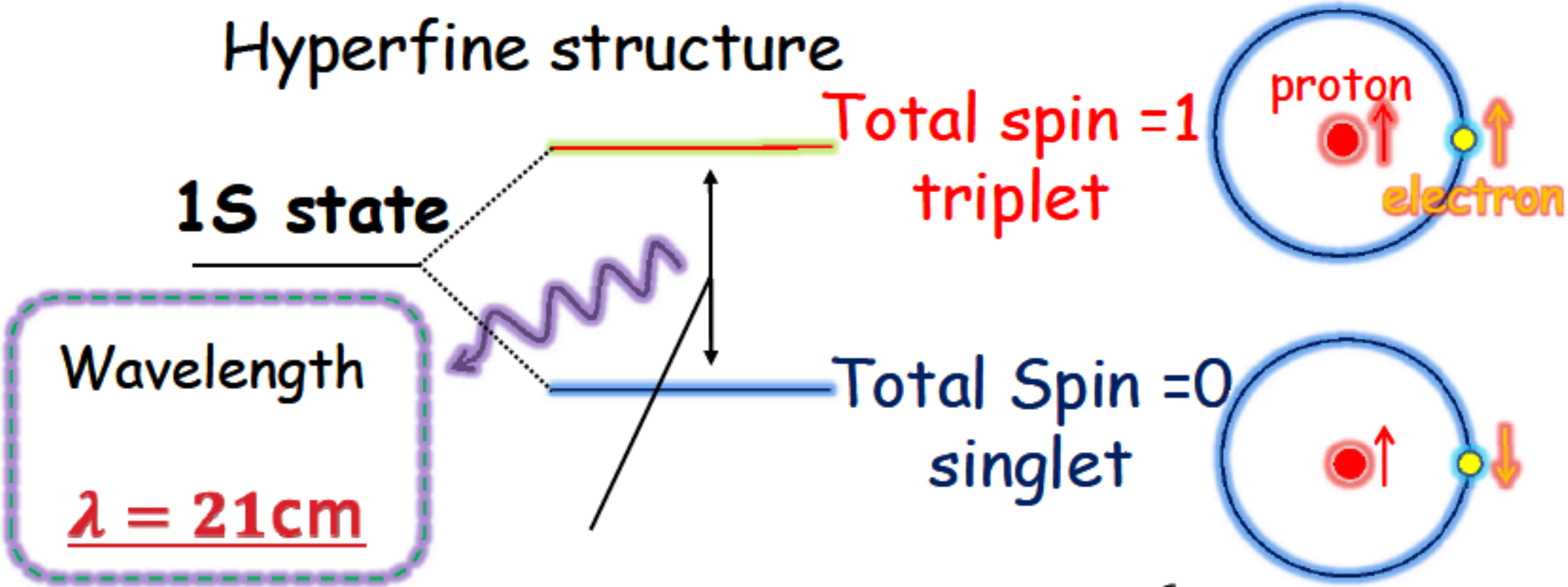
故宮博物院

Cosmological 21cm line observations

KK, Oyama, T.Takahashi, T.Sekiguchi, 2013

◇ 21cm line

◆proton-electron's spin-spin interaction



$$\Delta E = 2\pi\nu_{21} = 5.8 \times 10^{-6}\text{eV}$$

21 cm brightness temperature

$$\delta T_b^{obs}\left(\frac{\nu_{21}}{1+z},\boldsymbol{r},z\right) \approx A\frac{x_{HI}n_H}{(1+z)H(z)}\left[1-\frac{T_\gamma}{T_S}\right]\left[1-\boxed{\frac{1+z}{H(z)}\frac{d\mathrm{v}_{p||}}{dr}}\right]$$

Peculiar velocity

Optical depth
is small

$$1-e^{-\tau_{\nu_{21}}} \approx \tau_{\nu_{21}}$$

$$A \equiv \frac{3c^3\hbar A_{21}}{16\nu_{21}^2 k_B}$$

χ_{HI}:neutral fraction

Spin temperature

$$\frac{n_1}{n_0} \equiv \frac{g_1}{g_0}\exp\left(-\frac{h\nu_{21}}{k_B T_S}\right)$$

◇ 21cm brightness temperature fluctuation $\boldsymbol{\delta_{21}}$

$$\delta_{\partial v} \equiv \frac{1+z}{H(z)}\frac{dv_{p||}}{dr}$$

$$\delta_{21} \equiv \frac{\delta T_b^{obs} - \delta \bar{T}_b^{obs}}{\delta \bar{T}_b^{obs}}$$

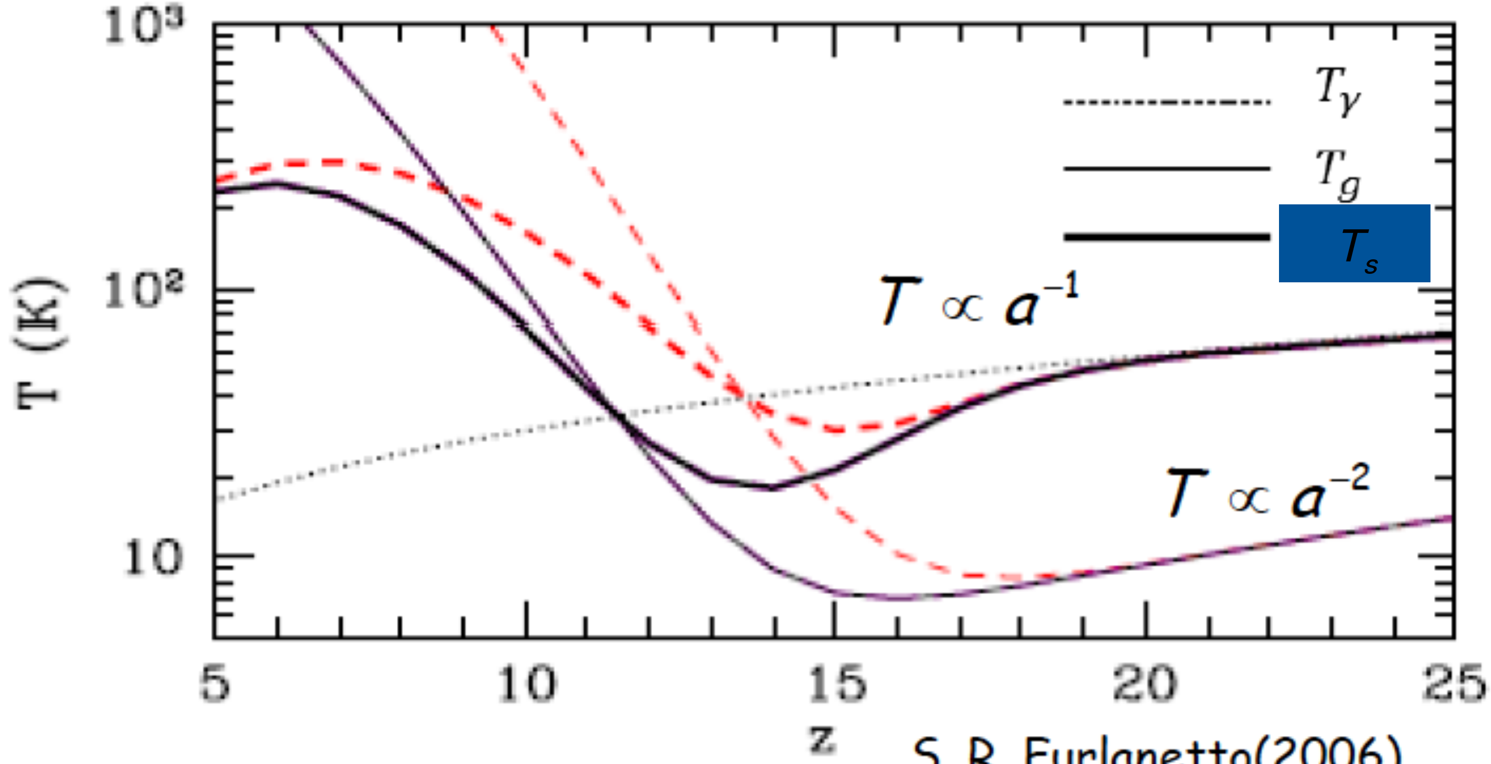
$$\delta_{21} = \left(\frac{1}{1+\delta_{T_S}}\right)\left[1 + \delta_H + \delta_{x_{HI}} + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_s} - \frac{\bar{T}_\gamma}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_\gamma} - \delta_{\partial v}\right.$$

$$+\left[\delta_{x_{HI}}\delta_H + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{x_{HI}}\delta_{T_s} - \frac{\bar{T}_\gamma}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_\gamma}\delta_{x_{HI}}\right.$$

$$\left. -\delta_{x_{HI}}\delta_{\partial v} + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{x_H}\delta_{T_s} - \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_s}\delta_{\partial v}\right] - 1$$

$\delta_{x_{HI}}, \delta_{T_s} \approx \mathcal{O}(1)$ になりえる

Evolution of spin temperature after star formation



S. R. Furlanetto(2006)

Gas was heated by X-ray emission

$$T_g > T_\gamma$$

$z \approx 10$ by Ly-α heating $T_S \to T_g \gg T_\gamma$

21cm line power spectrum $P_{21}(k,\mu)$

$$\langle \tilde{\delta}_{21}(\boldsymbol{k})\tilde{\delta}^*_{21}(\boldsymbol{k}') \rangle = (2\pi)^3 \delta^D(\boldsymbol{k} - \boldsymbol{k}')\, P_{21}(k,\mu)$$

M_ν < 0.1 eV $\delta_{x_{HI}} \ll 1$

$$P_{21}(k,\mu) = (1+\mu^2)^2 P_{\delta_H \delta_H}(k)$$

$P_{\delta_H \delta_H}(k)$: matter power spectrum

Detail of ionization history
gives us power spectrum

SKA (Square kilometer Array)

Location : Australia and South Africa

Antenna number

5000

Effective total Antenna area

$$6 \times 10^5 \ \mathrm{m}^2$$

http://www.skatelescope.org/

Construction Phase (2016 –)

Omniscope

Max Tegmark, Matias Zaldarriaga   arXiv:0805.4414v2 (2008)

Max Tegmark, Matias Zaldarriaga   Phys. Rev. D 82, 103501 (2010)

Lower cost than usual interferometers

- J. R. Pritchard, E. Pierpaoli, Phys Rev D 78, 065009

Antenna number	Effective total anntena area
10^6	10^6 m

21cm Fisher matrix

M.McQuinn, O.Zahn, M.Zaldarriaga, L.Hernquist, S.R. Furlanetto
(2006) Astrophys.J.653:815-830,2006

$$F_{ij} = \frac{1}{2}\sum_{i}^{N} \frac{1}{P_{T_b}^{tot}(k,\mu)^2} \frac{\partial P_{T_b}^{tot}(k,\mu)}{\partial \theta_i} \frac{\partial P_{T_b}^{tot}(k,\mu)}{\partial \theta_j}$$

$$P_{T_b}^{tot} \equiv \left(\delta \bar{T}_b^{obs}\right)^2 P_{21} + P_{Noise}$$

$$P_{Noise} \equiv \left(\frac{\lambda^2 T_{sys}}{A_e}\right)^2 \frac{1}{n_b t_0}$$ Detector Noise

CMB B-mode polarization

By Planck (ESA), PolarBEAR (USA,Japan), CMPpol (USA)

- Fisher matrices

$$\mathbf{F}_{ij}^{\mathrm{CMB}} = \sum_{l} \frac{(2\ell+1)}{2} f_{\mathrm{sky}} \times \mathrm{Trace}\left[\mathbf{C}_\ell^{-1}\frac{\partial \mathbf{C}_\ell}{\partial p_i}\mathbf{C}_\ell^{-1}\frac{\partial \mathbf{C}_\ell}{\partial p_j}\right]$$

$$\mathbf{C}_\ell = \begin{pmatrix} C_\ell^{\mathrm{TT}} + N_\ell^{\mathrm{TT}} & C_\ell^{\mathrm{TE}} & C_\ell^{\mathrm{Td}} \\ C_\ell^{\mathrm{TE}} & C_\ell^{\mathrm{EE}} + N_\ell^{\mathrm{EE}} & 0 \\ C_\ell^{\mathrm{Td}} & 0 & C_\ell^{\mathrm{dd}} + N_\ell^{\mathrm{dd}} \end{pmatrix}$$

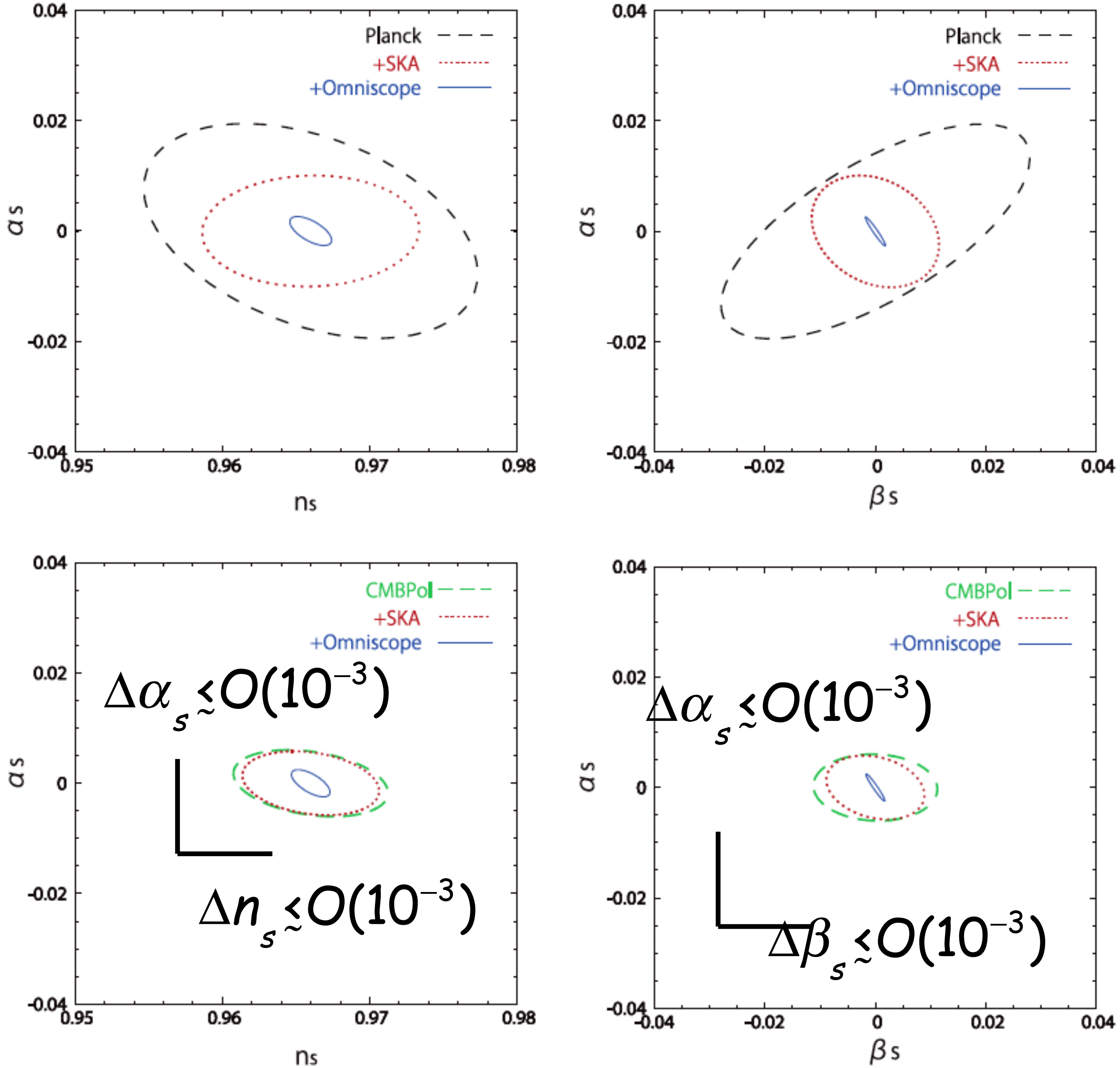
$$\mathbf{F}^{21\mathrm{cm}+\mathrm{CMB}} \simeq \mathbf{F}^{\mathrm{CMB}} + \mathbf{F}^{21\mathrm{cm}}$$

$$\mathbf{F}_{ij}^{21\mathrm{cm}} = \sum_{\mathrm{pixels}} \frac{1}{[\delta P_{21}(\mathbf{u})]^2}\left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_i}\right)\left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_j}\right)$$

Running and Running of

KK, Oyama, Sekiguchi, T.Takahashi (2013)

$$\alpha_s \equiv \frac{d \ln P_\zeta}{d \ln k}$$



$$\beta_s \equiv \frac{d\alpha_s}{d \ln k}$$

Sensitivities

$k_{\rm ref}$ $[\mathrm{Mpc}^{-1}]$	0.002	0.01	0.05	0.1	0.2	0.5
δn_s	3.81×10^{-3}	2.62×10^{-3}	5.53×10^{-4}	4.01×10^{-4}	4.68×10^{-4}	3.33×10^{-4}
$\delta \alpha_s$	1.47×10^{-3}	1.87×10^{-3}	1.00×10^{-3}	5.57×10^{-4}	$\boxed{2.64 \times 10^{-4}}$	6.65×10^{-4}
$\delta \beta_s$	2.43×10^{-4}	5.94×10^{-4}	6.86×10^{-4}	6.88×10^{-4}	6.87×10^{-4}	6.79×10^{-4}

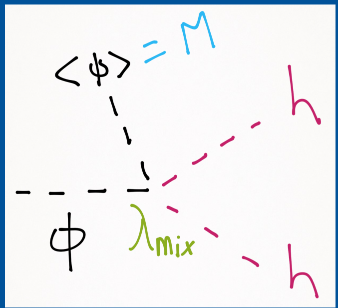
Table 5: Expected 1σ uncertainties of n_s, α_s and β_s from CMBpol+Omniscope for several values of $k_{\rm ref}$.

KK, Oyama, T.Takahashi, T.Sekiguchi, 2013

Reheating

- Decay

$$\Gamma_{\phi\to hh} \sim \frac{\lambda_{mix}^2}{16\pi}\frac{\langle\phi\rangle^2}{m_\phi} \sim 10^{-2}\frac{(\lambda_{mix}M)^2}{m_\phi} \sim 10^{-2}\frac{m_h^4}{m_\phi M^2}$$



$$\sim 10^{-14}GeV\left(\frac{10^9 GeV}{M}\right)^2$$

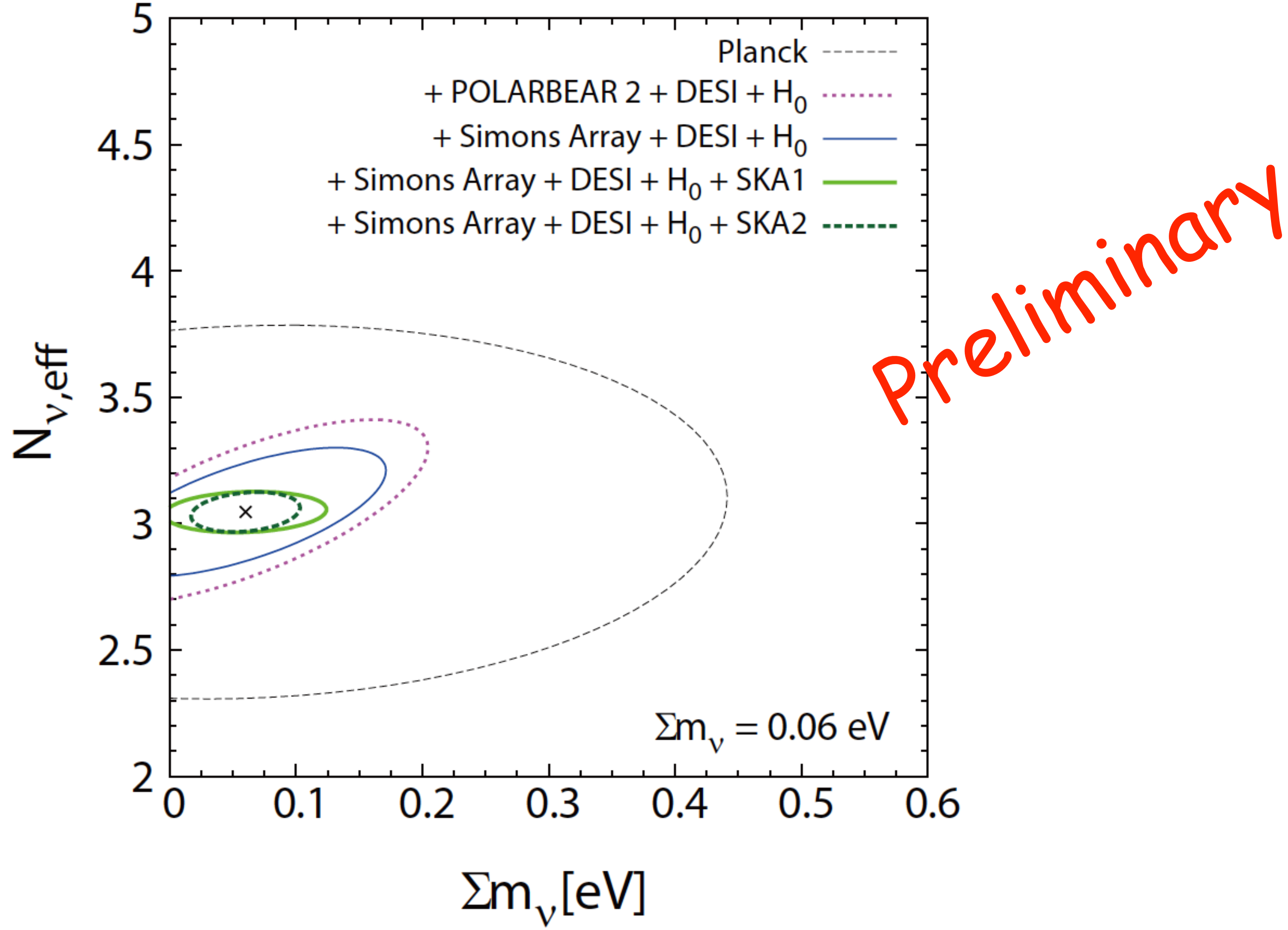
$$m_h = \sqrt{|\lambda_{\rm mix}|}M$$

- Reheating

$$\Gamma_{\phi\to hh} \equiv 3H(T_R)$$

$$T_R \sim 100GeV\left(\frac{10^9 GeV}{M}\right)$$

Future constraints on neutrino mass by 21cm, CMB, and BAO



Oyama, Kohri, Hazumi (2015) in preparation

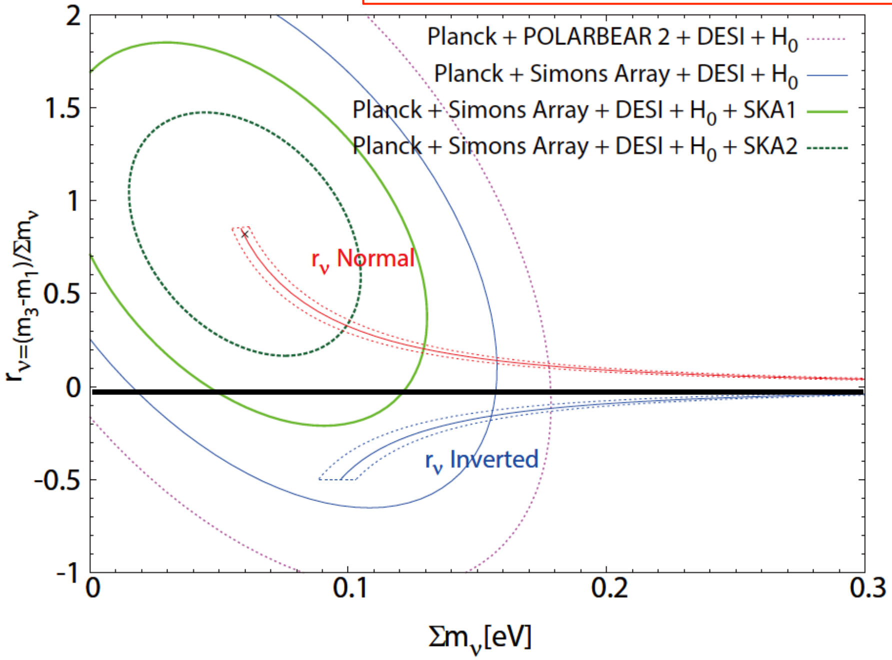
Future constraints on neutrino hierarchy by 21cm, CMB and BAO

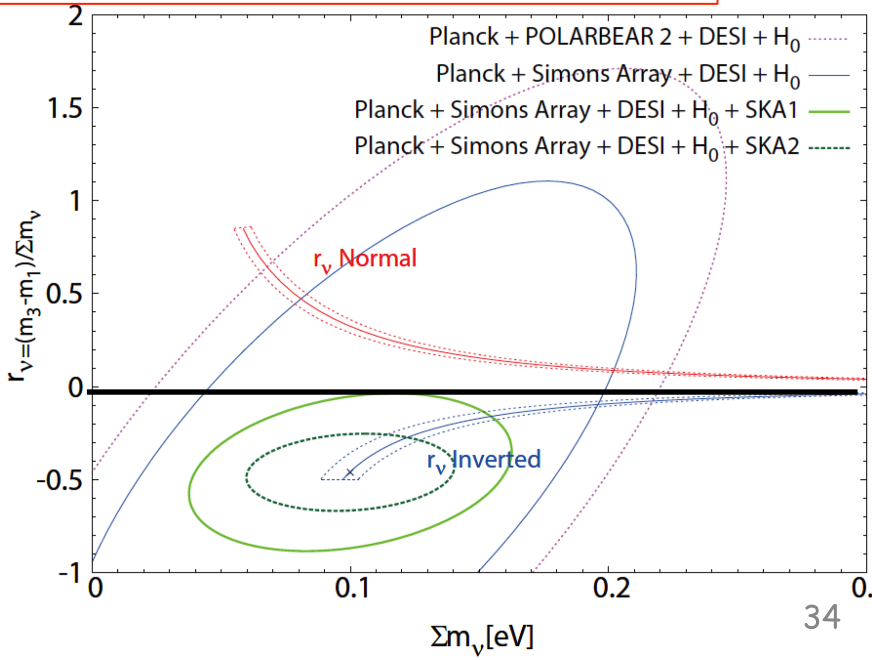
Oyama, Kohri, Hazumi (2015) in preparation

Preliminary

- Hierarchy parameter

$$r_\nu \equiv \frac{m_3 - m_1}{\Sigma m_i} = \begin{cases} > 0 & \text{normal hierarchy} \\ < 0 & \text{inverted hierarchy} \end{cases}$$





Conclusion

- Conformal infaltion models are attractive in terms of both large and small field inlflations
- After Planck's 2015 data release, we found that only the small-field model fits the observations

Outstanding issues

- The trajectory is not so trivial.
- Tensor to scalar ratio is small ($r \ll 10^{-20}$), which must not be observed forever
- A possible large-field preceding inflation could occur before the small-field CW conformal inflation