Schwinger Effect in (A)dS

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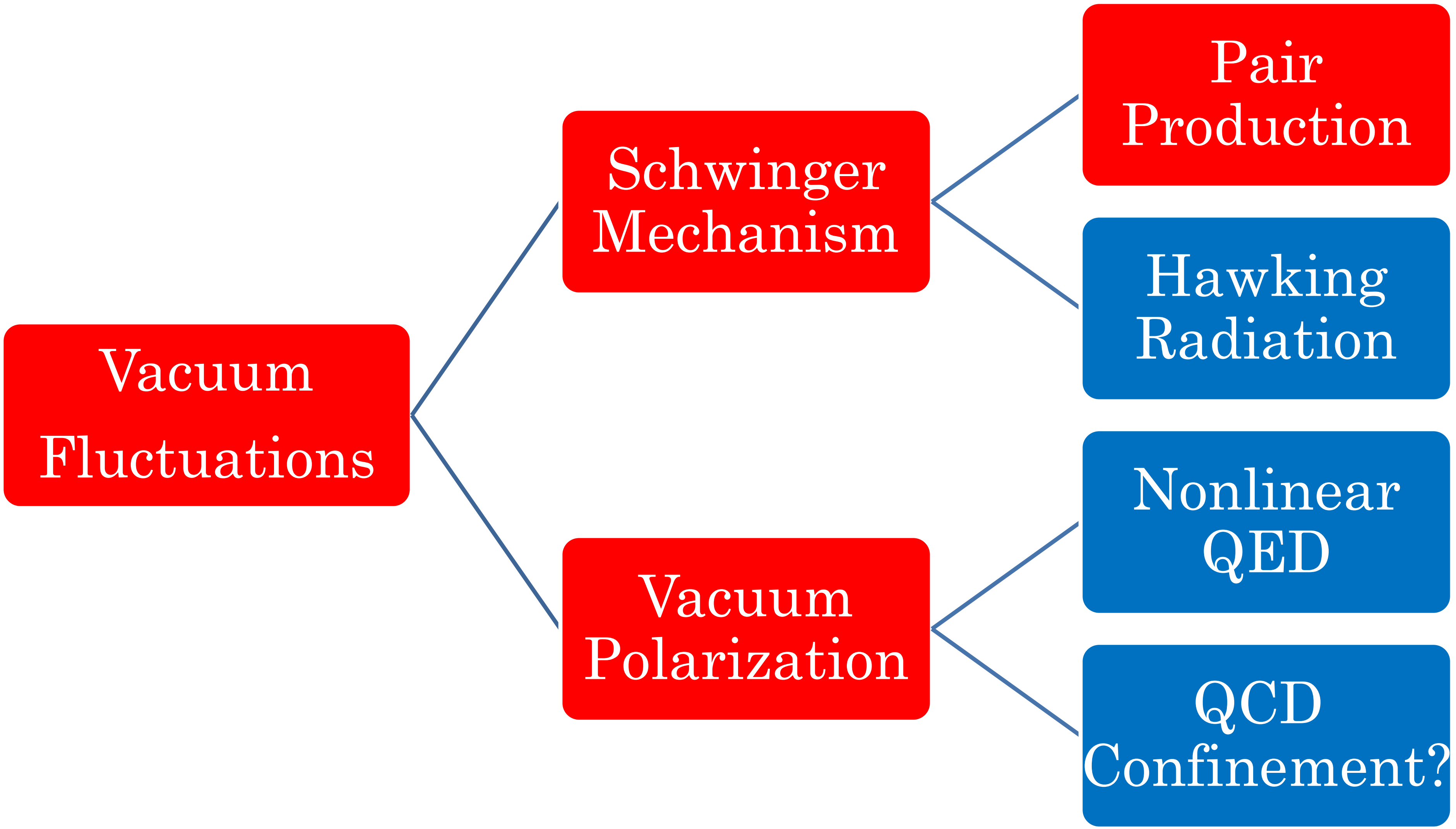
Outline of Lectures

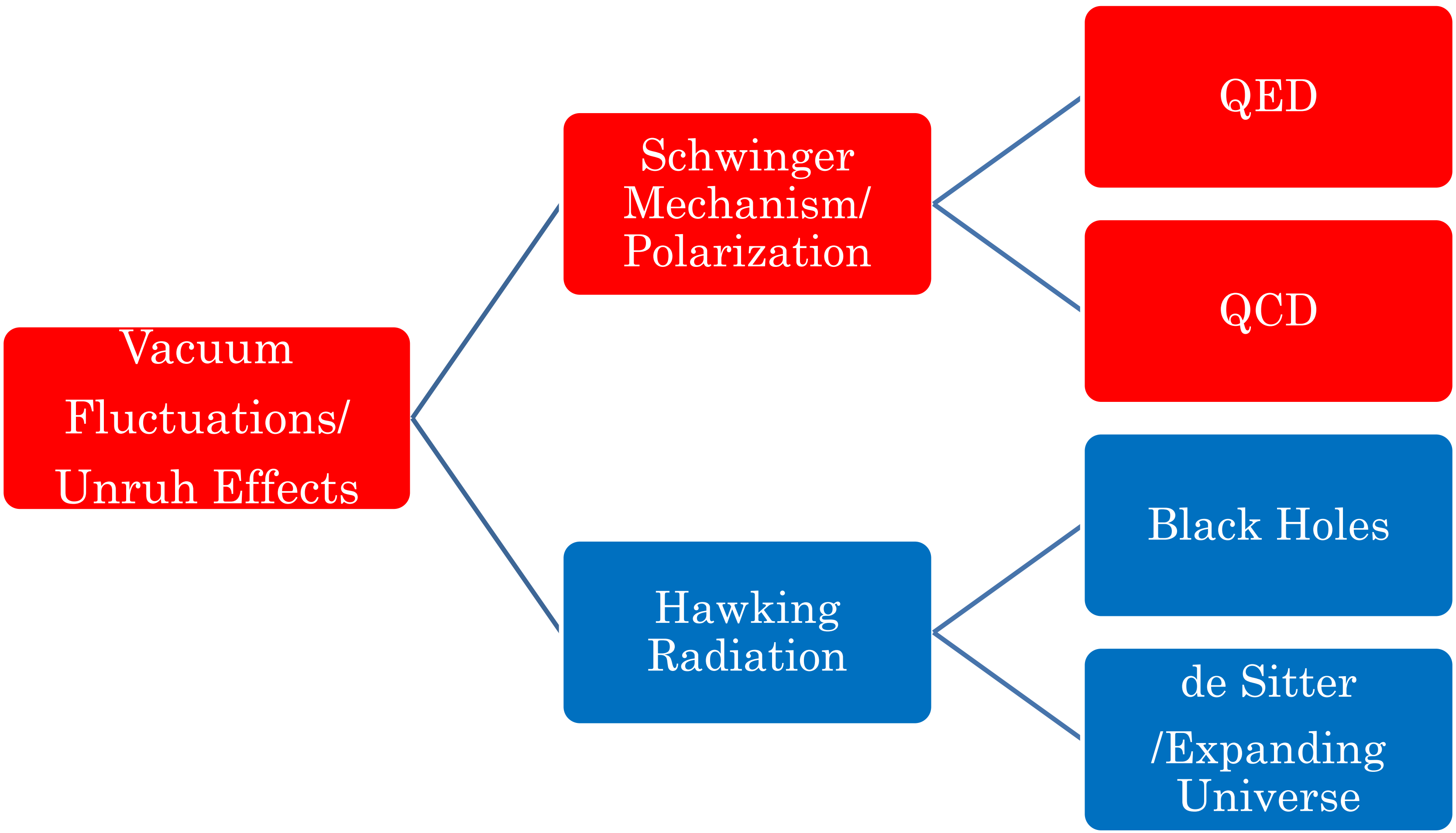
- (A)dS Space as Strong Field Physics
- Effective Actions in In-Out Formalism
- Quantum Field Theory in (A)dS Space
- Gibbons-Hawking's dS Radiation
- In-Out Formalism in dS Space
- Schwinger Effect in (A)dS
- Perspectives of Schwinger Effect in (A)dS

Physics in Strong Fields



Unified Picture for Pair Production

[SPK, JHEP11('07)]



Connecting Schwinger Mechanism & Unruh Effect & Hawking Radiation

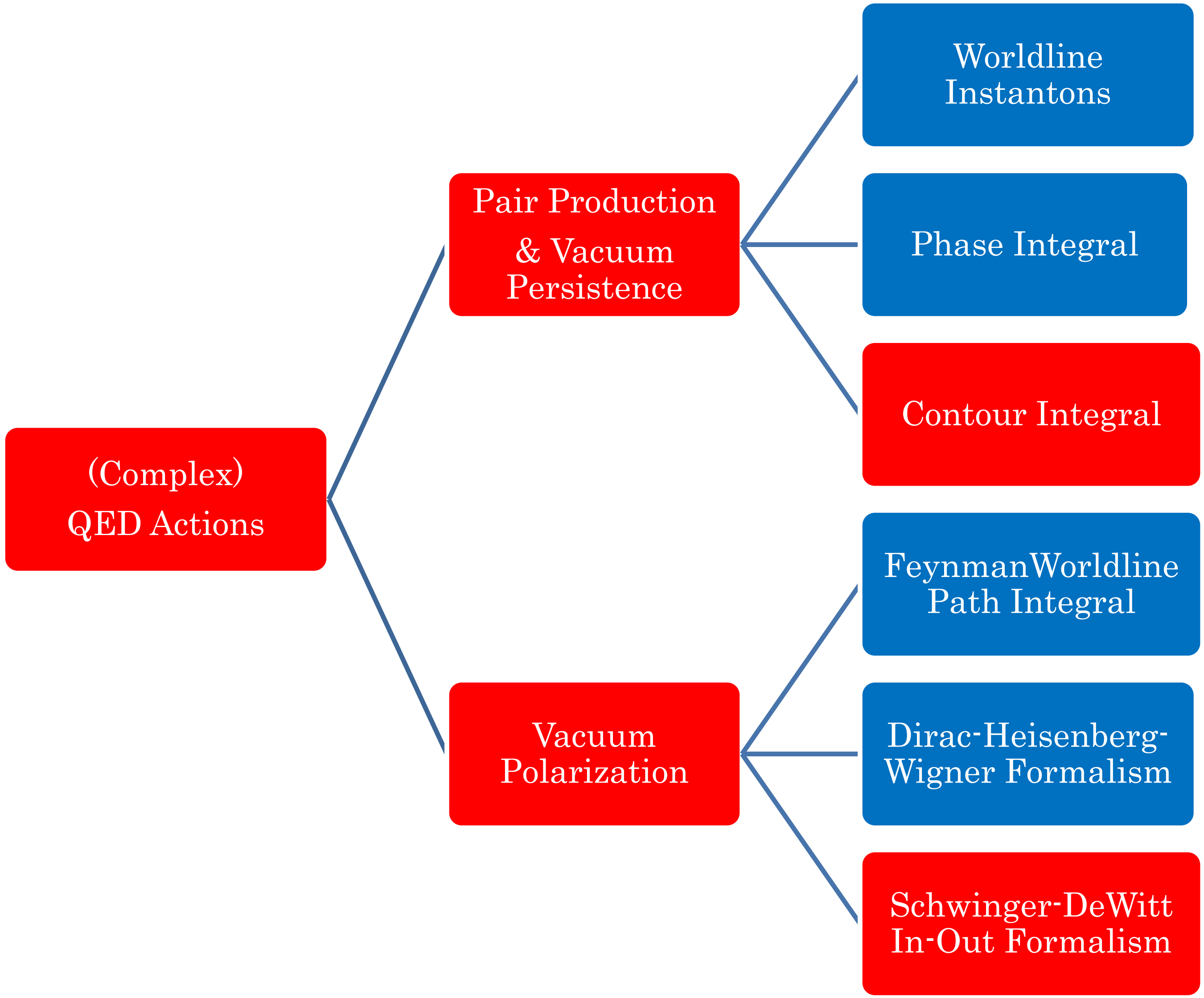
- An intuitive way to understand particle (pair) production

[Frolov & Novikov, *Black Hole Physics* ('98)]

$$P \cong \exp\left(-2\pi \frac{\text{energy (mass)}}{\text{force} \times \text{Compton wavelength}}\right) = \exp\left(-\frac{\text{energy (mass)}}{k_{\mathrm{B}} T}\right)$$

- Schwinger pair production: $F = eE\,,\ l = \dfrac{[\hbar]}{m[c]} \Leftrightarrow T = \dfrac{eE/m}{2\pi}$
- Unruh effect: $F = ma,\ l = \dfrac{[\hbar]}{m[c]} \Leftrightarrow T = \dfrac{a}{2\pi}$
- Hawking radiation: $F = m\kappa,\ l = \dfrac{[\hbar]}{m[c]} \Leftrightarrow T = \dfrac{\kappa}{2\pi}$
- dS (Gibbons-Hawking) radiation? $F = ma = m\kappa \Leftrightarrow T = \dfrac{H}{2\pi}$

Computing Effective Actions



What QED is in (A)dS?
Schwinger Effect in (A)dS?

Effective Actions in In-Out Formalism

: A Powerful and Consistent Method for QFT in Background Fields

In-Out Formalism for One-Loop Effective Actions

- In the in-out formalism, the vacuum persistence amplitude gives the effective action [Schwinger, PNAS ('51); DeWitt, Phys.Rep.19 ('75); *The Global Approach to Quantum Field Theory* ('03)] and is equivalent to the Feynman integral

$$e^{iW} = e^{i\int (-g)^{1/2} d^D x L_{\text{eff}}} = \langle 0, \text{out} \,|\, 0, \text{in} \rangle$$

- The complex effective action and the vacuum persistence for particle production

$$\left| \langle 0, \text{out} \,|\, 0, \text{in} \rangle \right|^2 = e^{-2\,\text{Im}W} \ , \quad 2\,\text{Im}W = \pm VT \sum_{\text{k}} \ln(1 \pm N_k)$$

Bogoliubov Transformation & In-Out Formalism

- The Bogoliubov transformation between the in-state and the out-state, equivalent to the S-matrix,

$$a_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} a_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} b^+_{\mathrm{k,in}} = U_{\mathrm{k}} a_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

$$b_{\mathrm{k,out}} = \alpha_{\mathrm{k,in}} b_{\mathrm{k,in}} + \beta^*_{\mathrm{k,in}} a^+_{\mathrm{k,in}} = U_{\mathrm{k}} b_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

- Commutation relations from quantization rule (CTP):

$$\left[a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p}),\ \left[b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right] = \delta(\mathrm{k}-\mathrm{p});$$

$$\left\{a_{\mathrm{k,out}}, a^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p}),\ \left\{b_{\mathrm{k,out}}, b^+_{\mathrm{p,out}}\right\} = \delta(\mathrm{k}-\mathrm{p})$$

- Particle (pair) production

$$N_{\mathrm{k}} = \left|\beta_{\mathrm{k}}\right|^2; \quad \left|\alpha_{\mathrm{k}}\right|^2 \mp \left|\beta_{\mathrm{k}}\right|^2 = 1$$

Out-Vacuum from In-Vacuum

- For bosons, the out-vacuum is the multi-particle states of but unitary inequivalent $\langle 0;\mathrm{out} \mid 0;\mathrm{in}\rangle = 0$ to the in-vacuum:

$$\left|0;\mathrm{out}\right\rangle = \prod_{\mathrm{k}} U_{\mathrm{k}} \left|0;\mathrm{in}\right\rangle = \prod_{\mathrm{k}} \frac{1}{\alpha_{\mathrm{k,in}}} \sum_{n_k} \left(-\frac{\beta^{*}_{\mathrm{k,in}}}{\alpha_{\mathrm{k,in}}} \right)^{n_k} \left| n_{\mathrm{k}}, \overline{n}_{\mathrm{k}}; \mathrm{in} \right\rangle$$

- The out-vacuum for fermions (Pauli blocking):

$$\left|0;\mathrm{out}\right\rangle = \prod_{\mathrm{k}} U_{\mathrm{k}} \left|0;\mathrm{in}\right\rangle = \prod_{\mathrm{k}} \left(-\beta^{*}_{\mathrm{k,in}} \left| 1_{\mathrm{k}}, \overline{1}_{\mathrm{k}}; \mathrm{in} \right\rangle + \alpha_{\mathrm{k,in}} \left| 0_{\mathrm{k}}, \overline{0}_{\mathrm{k}}; \mathrm{in} \right\rangle \right)$$

Out-Vacuum from S-Matrix

- The out-vacuum in terms of the S-matrix (evolution operator) for scalar

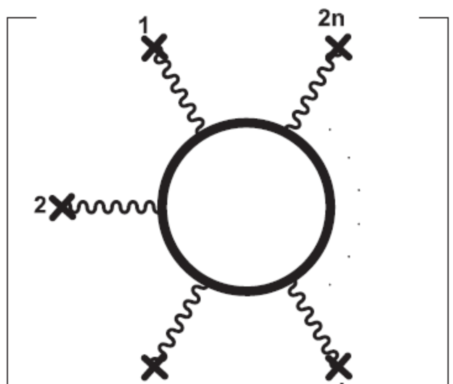
$$U_{\mathrm{k}} = S_{\mathrm{k}} P_{\mathrm{k}}, \quad \begin{cases} P_{\mathrm{k}} = \exp[i\theta_{\mathrm{k}}(a^{+}_{\mathrm{k,in}} a_{\mathrm{k,in}} + b^{+}_{\mathrm{k,in}} b_{\mathrm{k,in}} + 1)] \\ S_{\mathrm{k}} = \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}})] \end{cases}$$

$$\alpha_k = e^{-i\vartheta_k} \cosh r_k, \quad \beta^*_k = -e^{-i\vartheta_k}(e^{i2\varphi_k} \sinh r_k)$$

- The diagrammatic representation for pair production

$$|\mathrm{out}\rangle = \prod_k e^{-i\vartheta_k} \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2i\varphi_{\mathrm{k}}})] |\mathrm{in}\rangle$$

$$= \sum_{n=1}^{\infty}$$



Effective Actions at T=0 & T

- Zero-temperature effective actions in proper-time integral via gamma-function regularization [SPK, Lee, Yoon, PRD 78 ('08); 82 ('10); SPK, PRD 84 ('11)]; zeta-function regularization [SPK, Lee, arXiv:1406.4292]

$$W = -i \ln \langle 0, \mathrm{out} \mid 0, \mathrm{in} \rangle = \pm i \sum_{\mathrm{k}} \ln \alpha_{\mathrm{k}}^{*}$$

- finite-temperature effective action [SPK, Lee, Yoon, PRD 79 ('09); 82 ('10)]

$$\exp\left[i \int d^3 x dt L_{\mathrm{eff}} \right] = \langle 0, \beta, \mathrm{in} | U^{+} | 0, \beta, \mathrm{in} \rangle = \frac{\mathrm{Tr}(U^{+} \rho_{\mathrm{in}})}{\mathrm{Tr}(\rho_{\mathrm{in}})}$$

Vacuum Polarization & Persistence

- Effective action at T per unit volume per unit time

$$L_{\text{eff}} = \mp i \sum_{k,\sigma} \left[\ln\left(1 \pm e^{-\beta(\omega_k - z_k)}\right) \underbrace{-\beta z_k}_{\text{vacuum effective action}} \underbrace{-\ln\left(1 \pm e^{-\beta\omega_k}\right)}_{\text{zero field subtraction}} \right]$$

$$1/\alpha_k = e^{\beta z_k}, \quad z_k = z_r(k) + i z_i(k)$$

- Purely thermal part of the effective action

$$\begin{aligned} \Delta L_{\text{eff}}(T,E) &= L_{\text{eff}}(T,E) - L_{\text{eff}}(T=0,E) \\ &= \mp i \sum_{k,\sigma} \left[\ln\left(1 \pm e^{-\beta(\omega_k - z_k)}\right) - \ln\left(1 \pm e^{-\beta\omega_k}\right) \right] \end{aligned}$$

Vacuum Polarization & Persistence

- Imaginary part of the effective action (vacuum persistence) at T

$$\mathrm{Im}(\Delta L_{\mathrm{eff}}) = \pm\frac{1}{2} i \sum_{k,\sigma} \sum_{j=1} \frac{(\mp n_{FD/BE}(k))^j}{j} \left[(e^{\beta z_k} - 1)^j + (e^{\beta z_k^*} - 1)^j \right]$$

- Real part of the effective action (vacuum polarization) at T

$$\mathrm{Re}(\Delta L_{\mathrm{eff}}(T)) = \mp \sum_{k,\sigma} \arctan \left[\frac{\sin(\mathrm{Re}\, L_{\mathrm{eff}}(T=0,k))}{e^{\beta\omega_k} \left(1 + |\beta_k|^2\right)^{(1+2|\sigma|)/2} \pm \cos(\mathrm{Re}\, L_{\mathrm{eff}}(T=0,k))} \right]$$

Quantum Field Theory in (A)dS

de Sitter Space Still Interesting?

- The present and future universe dominated by Dark Energy will be described by an asymptotically dS space.
- The pure dS with the cosmological constant Λ has a cosmological horizon and emits the Gibbons-Hawking's dS radiation and the Stokes phenomenon occurs for radiation.
- The maximal symmetry of dS as for Minkoswki spacetime. BUT we still do NOT comprehend the vacuum structure of dS space. The Bunch-Davies vacuum not satisfying Feynman's composition rule challenged by Polyakov [Polyakov, NPB834(2010); NPB797(2008)].

Why QED in (A)dS?

- The radial motion of a scalar wave in the Nariai geometry of a rotating black hole is the dS space is equivalent to a massive charge in a uniform electric field in dS_2 [Anninos, Hartman, JHEP03 ('10); Anninos, Anous, JHEP08 ('10)].
- The near-horizon geometry of an extremal rotating black hole or an extremal Reissner-Nordstrom black hole is equivalent to that in the uniform electric field in AdS_2 [Barden, Horowitz, PRD60 ('99); Chen et al, PRD85 ('12)].
- Thermal interpretation of the Schwinger formula for pair production in QGP and AdS/CFT.

One-Loop Gauge/Gravity Relation

- Massive scalar in a self-dual EM field in 4d-D = massive spinor in 2d-D AdS at one-loop [Basar, Dunne, JPA 43 ('10)]

$$\text{gauge} \Leftrightarrow \text{gravity}$$

$$D = 4d \Leftrightarrow D = 2d$$

$$\text{massive scalar} \Leftrightarrow \text{massive spinor}$$

$$\text{maximally symmetric gauge field} \Leftrightarrow \text{maximally symmetric gravitational curvature}$$

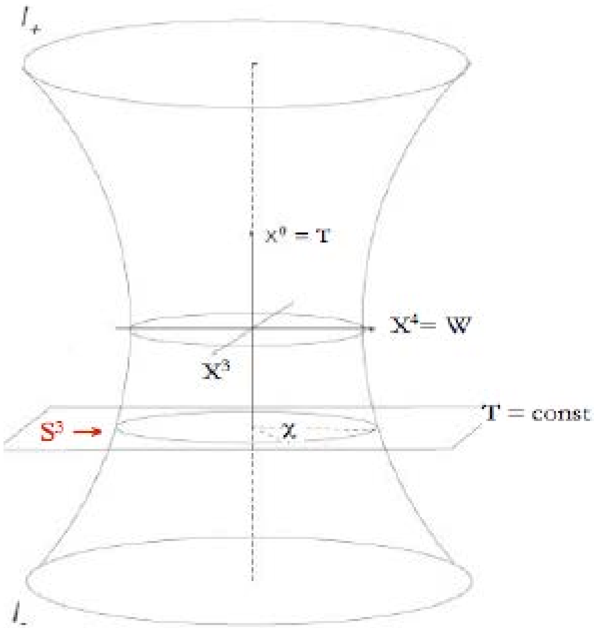
$$\frac{\mathrm{m}^2}{2f} \Leftrightarrow \sqrt{\frac{m^2}{R}}$$

- The Heisenberg-Euler/Schwinger effective actions in QED are better understood in the weak and strong field regime.

Gibbons-Hawking's dS Radiation

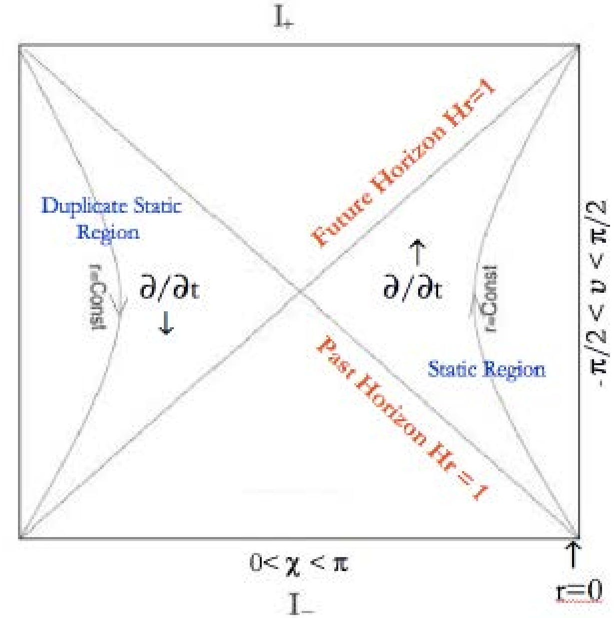
de Sitter Space

Embedded hyperboloid



$$ds^2 = -dt^2 + \frac{\cosh^2(Ht)}{H^2} d\Omega_3^2$$

(Carter-)Penrose Diagram



$$ds^2 = -(1-(Hr)^2)dt^2 + \frac{dr^2}{1-(Hr)^2} + r^2 d\Omega_2^2$$

[E. Mottola, arXiv:1008.5006]

dS Space as a Black Hole?

- Gibbons and Hawking [PRD15('77)]: a dS space has a surface gravity $\kappa = H$ at the cosmological horizon $r = 1/H$, the temperature $T = H/(2\pi)$ and the entropy $S = A/4$.
- Jacobson [PRL75('95)]: the black hole thermodynamics seen by all local Rindler observers is equivalent to the Einstein equation [equilibrium condition].
- Cai and Kim [JHEP02('05)]: the black hole thermodynamics at the apparent horizon of an expanding FRW universe is equivalent to the Friedmann equation [nonequilibrium condition].
- An open question is, "what is the origin of the cosmological entropy?"

dS Radiation

- The the Bunch-Davies vacuum for a massive scalar in the planar and the global coordinates

$$\begin{cases} \varphi_k(t) = \left(\dfrac{\pi}{4H}\right)^{1/2} e^{-3Ht/2} H_{ip}^{(1)}\left(\dfrac{ke^{-Ht}}{H}\right), \quad p = \left(\dfrac{m^2}{H^2} - \dfrac{9}{4}\right)^{1/2} \\ \varphi_k(t) = \left(\dfrac{2^{2l+3}}{2Hp}\right)^{1/2} \cosh^l(Ht) e^{(l+3/2-ip)Ht} F\left(l+\dfrac{3}{2}, l+\dfrac{3}{2}-ip; 1-ip; -e^{2Ht}\right) \end{cases}$$

- The in-vacuum and the out vacuum solutions

$$\begin{cases} \varphi_k^{(\mathrm{in})}(t) = \dfrac{e^{-Ht}}{\sqrt{2k}} \exp\left(\dfrac{ke^{-Ht}}{H}\right) \\ \varphi_k^{(\mathrm{in})}(t) = \dfrac{e^{-3Ht/2}}{\sqrt{2Hp}} e^{-iHpt} \end{cases} \qquad \varphi_k^{(\mathrm{out})}(t) = \dfrac{e^{-3Ht/2}}{\sqrt{2Hp}} e^{-iHpt}$$

dS Radiation

- The Bogoliubov transformation between the in-vacuum and the out-vacuum solutions

$$\varphi_k^{(\mathrm{BD})}(t) = \alpha_k \varphi_k^{(\mathrm{out})}(t) + \beta_k \varphi_k^{(\mathrm{out})*}(t)$$

- The Bogoliubov coefficients and pair production in the planar coordinates and the global coordinates

$$\begin{cases} N_k = \dfrac{1}{e^{2\pi p/H} - 1} \\ N_k = \dfrac{1}{\sinh^2(\pi p)} \end{cases} \qquad p = \left(\frac{m^2}{H^2} - \frac{9}{4} \right)^{1/2}$$

dS Radiation as Tunneling

- The in-out formalism ($t = \pm\infty$) predicts particle production only in even dimensions [Mottola, PRD 31 ('85); Bousso et al, PRD 65 ('02)].
- A massive scalar in a dS_{d+1} space:

$$\Phi(t,\Omega) = a^{-d/2}(t)\sum_k u_k(\Omega)\phi_k(t); \quad a = \frac{\cosh(Ht)}{H}$$

$$\nabla^2 u_k(\Omega) = -k^2 u_k(\Omega); \qquad k^2 = l(l+d-1)$$

$$\ddot{\phi}_k(t) + Q_k(t)\phi_k(t) = 0$$

$$Q_k(t) = m^2 + \frac{k^2}{a^2} - \frac{d(d-2)}{4}\left(\frac{\dot{a}}{a}\right)^2 - \frac{d}{2}\frac{\ddot{a}}{a}$$

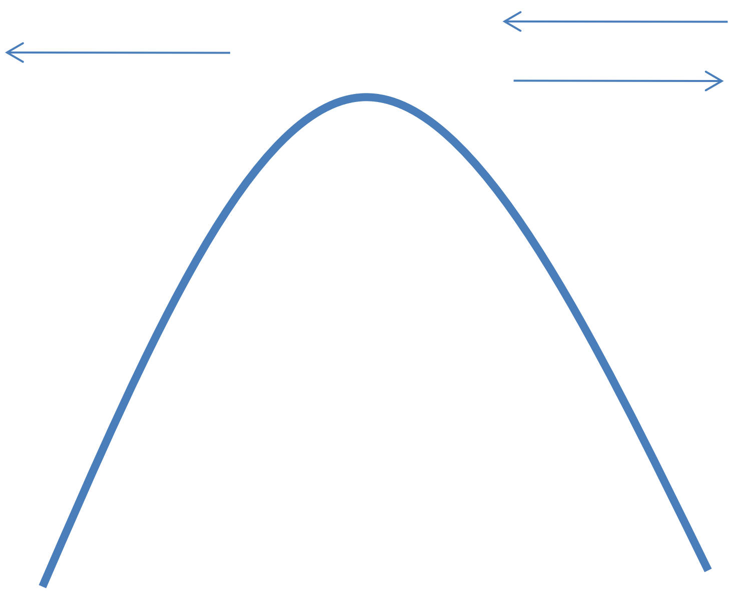
dS Radiation as Tunneling

- The Hamilton-Jacobi action in a complex time

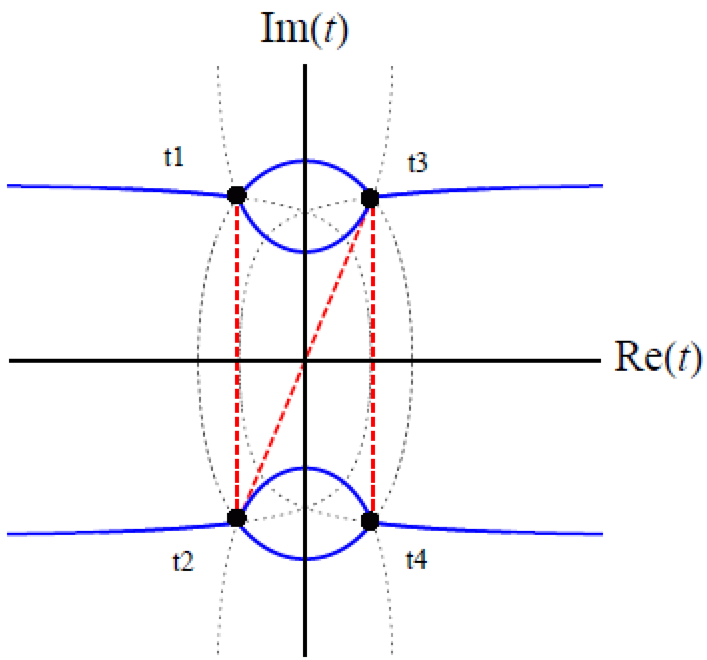
$$\phi_k(t) = e^{-iS_k(t)};\ \ S_k(t) = \int \sqrt{Q_k(z)}\,dz;\ \ Q_k(t) = p^2 + \frac{(\lambda H)^2}{\cosh^2(Ht)}$$

$$p = \sqrt{m^2 - (dH/2)^2};\ \lambda^2 = l(l+d-1) - d(d-2)/4$$

$$\Gamma_k = |\phi_k(t)|^2 = e^{-2\,\mathrm{Im}\,S_k(t)}$$



Stokes Phenomenon



[Fig. adopted from Dumlu & Dunne, PRL 104 (2010)]

- Two pairs of turning points

$$e^{Ht_{(a)\pm}} = -i\frac{\lambda H}{p} \pm i\sqrt{\frac{(\lambda H)^2}{p^2}+1}$$

$$e^{Ht_{(b)\pm}} = +i\frac{\lambda H}{p} \pm i\sqrt{\frac{(\lambda H)^2}{p^2}+1}$$

- Hamilton-Jacobi action

$$S_k(t_{(a)\pm}, t_{(b)\pm}) = i\pi\frac{p}{H} + \pi\lambda$$

dS Radiation as Tunneling

- Use the phase-integral approximation and find the mean number of produced particles [SPK, JHEP09('10)].

$$\overline{N}_k = e^{-2\,\mathrm{Im}\,S(I)} + e^{-2\,\mathrm{Im}\,S(II)} + 2\cos(\mathrm{Re}\,S(I,II))e^{-\mathrm{Im}\,S(I)-\mathrm{Im}\,S(II)}$$

$$\approx 4\sin^2(\pi(l+d/2))e^{-2\pi p/H}$$

$$\overline{N}_k = \frac{\sin^2(\pi(l+d/2))}{\sinh^2(2\pi p/H)}$$

- The Stokes phenomenon explains the destructive interference between two Stokes's lines in odd dimensions and the constructive interference in even dimensions [solitonic character].

In-Out Formalism for dS Radiation

Effective Action for dS

- de Sitter space with the metric

$$ds^2 = -dt^2 + \frac{\cosh^2(Ht)}{H^2} d\Omega_d^2$$

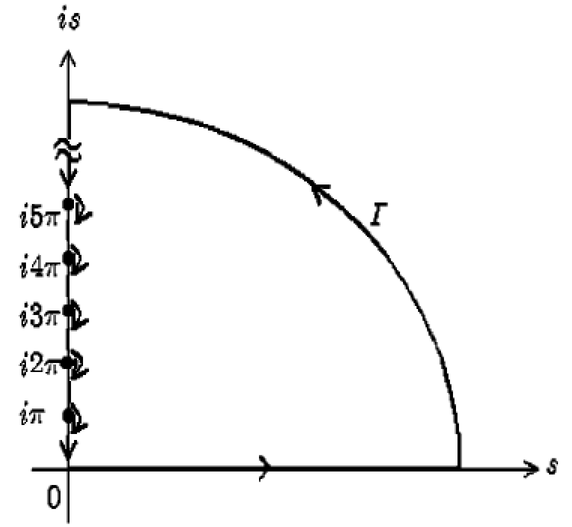
- Bogoliubov coefficients

$$\alpha_l = \frac{\Gamma(1-i\gamma)\Gamma(-i\gamma)}{\Gamma(l+d/2-i\gamma)\Gamma(1-l-d/2-i\gamma)}, \qquad l \in Z^0$$

$$\beta_l = \frac{\Gamma(1-i\gamma)\Gamma(i\gamma)}{\Gamma(l+d/2)\Gamma(1-l-d/2)}, \qquad \gamma = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}}$$

Effective Action for dS

- The Gamma-function regularization and the residue theorem
- The effective action per Hubble volume and per Compton time



$$L_{\text{eff}}(H) = \frac{\Gamma(\frac{d+1}{2}) m H^d}{(2\pi)^{(d+1)/2}} \sum_{l=0}^{\infty} D_l^{(d)} P \int_0^{\infty} ds \frac{e^{-\gamma s}}{s} \left[\frac{\cos((2l+d-1)s/2) - \cos(s/2)}{\sin(s/2)} \right]$$

$$2 \operatorname{Im} L_{\text{eff}}(H) = \ln\left(1 + \overline{N}_l\right), \quad \overline{N}_l = |\beta_l|^2 = \left(\frac{\sin \pi(l + d/2)}{\sinh(\pi\gamma)} \right)^2$$

One-Loop Effective Action for dS

- Bosonic vacuum polarization [SPK, arXiv:1008.0577]

$$\bar{N}_l^{\rm sc} = |\beta_l|^2 = \left(\frac{\sin \pi(l + d/2)}{\sinh(\pi\gamma)} \right)^2, \quad 2\,{\rm Im}\, L_{\rm eff}^{\rm sc}(H) = \ln\left(1 + \bar{N}_l^{\rm sc}\right)$$

$$W_{\rm eff}^{\rm sc}(H) = \sum_{\rm states} P\int_0^\infty ds \frac{e^{-\gamma s}}{s} \left[\frac{\cos((2l + d - 1)s/2) - \cos(s/2)}{\sin(s/2)} \right]$$

- Fermionic vacuum polarization

$$\bar{N}_\lambda^{\rm sp} = |\beta_\lambda|^2 = \left(\frac{\sin \pi\lambda}{\cosh(\pi m/H)} \right)^2, \quad 2\,{\rm Im}\, L_{\rm eff}^{\rm sp}(H) = -\ln\left(1 - \bar{N}_\lambda^{\rm sp}\right), \ \lambda = \text{Dirac spectrum on } S^3$$

$$W_{\rm eff}^{\rm sp}(H) = -2\sum_{\rm states} P\int_0^\infty ds \frac{e^{-ms/H}}{s} \left[\frac{\sin^2(\lambda s/2)}{\sin(s/2)} \right]$$

No Quantum Hair for dS Space?

- The effective action per Hubble volume and per Compton time, for instance, in D=4

$$L_{\text{eff}}(H) = \frac{mH^3}{(2\pi)^2} \sum_{l=0}^{\infty} (l+1)^2 P \int_0^{\infty} ds \frac{e^{-\gamma s}}{s} \left[\frac{\cos((l+1)s) - \cos(s/2)}{\sin(s/2)} \right]$$

- Zeta-function regularization [Hawking, CMP 55 ('77)]

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}, \quad \zeta(-2n) = 0,\ n \in Z^+,\ \zeta(0) = -\frac{1}{2}$$

$$L_{\text{eff}}(H) = 0$$

κ-regularization

- The κ–regularization for angular momentum sum [SPK, arXiv:1102.4154]

$$\kappa(s,n)=\sum_{l=1}^{\infty} l^n \cos(ls)=\frac{1}{2\Gamma(-n)}\int_0^{\infty} ds\,\frac{dt}{t^{n+1}}e^{-t}\left[\frac{e^t\cos(s)-1}{\cosh(t)-\sin(s)}\right]$$

- Use the gamma function Γ(0) = -Γ(1) = 2Γ(-2) = ∞ and the 'possible' nonvanishing terms add up to zero:

$$\kappa(s,2)=-\frac{1}{2}-\frac{\cos(s)}{1-\cos(s)}+\frac{1+\cos(s)}{2(1-\cos(s))}=0$$

Schwinger Effect in (A)dS

Schwinger formula in (A)dS

- (A)dS metric and the gauge potential for E

$$ds^2 = -dt^2 + e^{2Ht}dx^2\,, \quad A_1 = -(E/H)(e^{Ht} - 1)$$

$$ds^2 = -e^{2Kx}dt^2 + dx^2\,, \quad A_0 = -(E/K)(e^{Kx} - 1)$$

- Schwinger formula in (A)dS: N = e^{-S} [SPK, Page PRD78 ('08)]

$$S_{dS} = \frac{2\pi}{H}\left[\sqrt{\left(\frac{qE}{H}\right)^2 + m^2 - \frac{H^2}{4}} - \frac{qE}{H}\right]$$

$$S_{AdS} = \frac{2\pi}{K}\left[\frac{qE}{K} - \sqrt{\left(\frac{qE}{K}\right)^2 - m^2 - \frac{K^2}{4}}\right]$$

Effective temperature for Schwinger formula

- Effective temperature for accelerating observer in (A)dS [Narnhofer et al, IJMP B10 (‘96); Deser, Levin, CQG14 (‘97)]

$$N = e^{-m/T_{\text{eff}}}\ ,\ \ T_{\text{eff}} = \sqrt{T_{\text{U}}^2 + \frac{R}{8\pi^2}},\ \ R = \pm 2H^2(K^2)$$

- Effective temperature for Schwinger formula in (A)dS [Cai, SPK, JHEP09 (‘14)]

$$N = e^{-\overline{m}/T_{\text{eff}}}\ ,\ \ \overline{m} = \sqrt{m^2 - \frac{R}{8}},\ \ T_{\text{U}} = \frac{qE}{m},\ \ T_{\text{GH}} = \frac{H}{2\pi}$$

$$T_{\text{dS}} = \sqrt{T_{\text{U}}^2 + T_{\text{GH}}^2} + T_{\text{U}}\ ;\ \ T_{\text{AdS}} = \sqrt{T_{\text{U}}^2 + \frac{R}{8\pi^2}} + T_{\text{U}}$$

One-Loop Action for dS in E

- Pair production and vacuum polarization [Cai, SPK, JHEP09 (‘14)]

$$N_{\mathrm{dS}} = \frac{e^{-(S_\mu - S_\lambda)} + e^{-2S_\mu}}{1 - e^{-2S_\mu}}, \quad 2\,\mathrm{Im}\,W_{\mathrm{dS}}^{(1)} = \ln\left(1 + N_{\mathrm{dS}}\right)$$

$$W_{\mathrm{dS}}^{(1)} = \frac{qE}{2\pi} P\int_0^\infty \frac{ds}{s}\left[e^{-(S_\mu - S_\lambda)s/2\pi}\left(\frac{1}{\sin(s/2)} - \frac{2}{s} - \frac{s}{12}\right) - e^{-S_\mu s/\pi}\left(\frac{\cos(s/2)}{\sin(s/2)} - \frac{2}{s} + \frac{s}{6}\right)\right]$$

$$S_\mu = \frac{2\pi}{H}\sqrt{\left(\frac{qE}{H}\right)^2 + m^2 - \frac{H^2}{4}}, \quad S_\lambda = \frac{2\pi}{H}\frac{qE}{H}$$

One-Loop Action for AdS in E

- Pair production and vacuum polarization [Cai, SPK, JHEP09 ('14)]

$$N_{\mathrm{AdS}} = \frac{e^{-(S_\kappa - S_\nu)} - e^{-(S_\kappa + S_\nu)}}{1 + e^{-(S_\kappa + S_\nu)}}, \quad 2\,\mathrm{Im}\,W^{(1)}_{\mathrm{AdS}} = \ln\left(1 + N_{\mathrm{AdS}}\right)$$

$$W^{(1)}_{\mathrm{AdS}} = -\frac{qE}{2\pi} P \int_0^\infty \frac{ds}{s} e^{-S_\kappa s/2\pi} \cosh(S_\nu s/2\pi) \left[\frac{1}{\sin(s/2)} - \frac{2}{s} - \frac{s}{12}\right]$$

$$S_\nu = \frac{2\pi}{K}\sqrt{\left(\frac{qE}{K}\right)^2 - m^2 - \frac{K^2}{4}}, \quad S_\kappa = \frac{2\pi}{K}\frac{qE}{K}$$

Perspectives of QED in (A)dS

- Black hole physics
  - (near) extremal limits of rotating black holes
- AdS/CFT
  - Thermalization of QGP
  - Schwinger effect in chromo-electromagnetic fields may have a thermal interpretation (Unruh temperature)
- "ER=EPR" (Einstein-Rosen bridge = Einstein Podolsky Rosen
  - Schwinger pair is a perfect entangled state (particle at one end of wormhole and antiparticle at the other.