The Singlet Extension of Supersymmetric Standard Model with Peccei-Quinn Symmetry

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Introduction

- Particle physics after discovery of the 125GeV Higgs boson?

 - Experiments/observations show that it is much like what predicted from the Standard Model with m=125GeV, λ(m_t)=0.12 based on electroweak symmetry breaking: No new physics discovered yet.

 - Is it the end of particle physics? : I don’t think so.

- Fortunately, the Standard Model itself is incomplete:

 We just have writeen down the Higgs potential but we do not know either the mechanism how the shape of the potential looks like a Mexican hat or why v should be at `the sclae’, 246GeV

$$V(h) = \frac{\lambda}{4}(h - v^2)^2$$

- Moreover, for particle physics to describe Nature perfectly(?), there are many phenomena that the Standard Model cannot accommodate:

0. The microscopic description of gravity
1. Non baryonic dark matter
2. Dark energy accelerating the Universe
3. The origin of extremely tiny neutrino mass
4. Inflation model resolving causality problem in co∟mology
5. The origin of baryon asymmetry

Therefore we need a new physics beyond the Standard Model.

Problem: we do not know any characteristic energy scale for a new physics describing them.

Hopefully they may be low enough so that we can probe them at LHC.

- Traditional story: Regarding the naturalness as a guiding principle, new physics should appear at LHC.

Naturalness

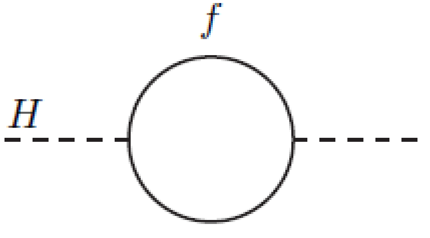
- Dimensionless couplings or a ratio of dimensionful parameters are order one, then they are natural.

- t'Hooft's technical naturalness: if such quantities are tiny, then we have an enhanced symmetry in its vanishing limit.

- Relating naturalness and the origin of the electroweak symmetry breaking(EWSB) scale:

  - This requires much high energy cutoff scale (grand unification scale, Planck mass scale….) to convert the problem to asking ``why EWSB scale is much smaller than such cutoff scale?”

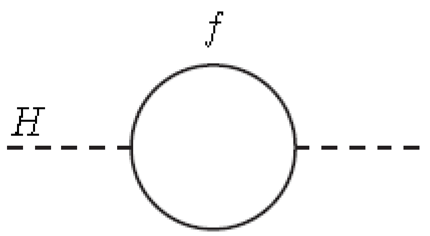
  - The existence of the fundamental scalar, Higgs at EWSB scale implies that the naturalness is worth to visit: The low scale mass of a fundamental scalar is unstable against quadratic divergent quantum correction.

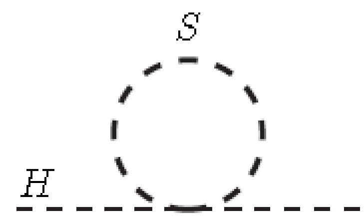


$$\delta m_h^2 = -\frac{|y_f|^2}{8\pi^2}\Lambda_{\mathrm{UV}}^2 + \cdots$$

- Supersymmetry(SUSY) resolves such quantum instability in an elegant way.

- In the supersymmetric limit, the cancellation between bosonic and fermionic loop occurs.





$$\delta m_h^2 = -\frac{|y_f|^2}{8\pi^2}\Lambda_{\rm UV}^2 + \frac{|y_f|^2}{8\pi^2}\Lambda_{\rm UV}^2$$

- If the SUSY breaking scale is responsible for the electroweak symmetry breaking, we expect milder behavior of the scalar mass quantum correction.

$$\delta m_h^2 = \frac{|y_f|^2}{8\pi^2} m_{\rm SUSY}^2 \log\frac{\Lambda_{\rm UV}}{\mu}$$

- So, we expect that the EWSB is generated from the SUSY breaking effect and its scale is determined by the conspiracy between SUSY breaking parameters (cf. mu problem : parameters at the SUSY breaking scale.

- In this sense, the most natural scenario is :

SUSY breaking scale $\sim \mathcal{O}(100)\mathrm{GeV}$

which is ruled out after LEP run.

- So we should accept the `little hierarchy' between SUSY breaking scale and EWSB scale.
- LHC run has ruled out new physics scale including SUSY breaking scale up to TeV (about 1% fine tuning).

Viable SUSY Model?

- Lessons from LHC run:

  - SUSY breaking scale allows at least 1% fine tuning. (can be worse if LHC fails to find SUSY)

  - If we are to keep naturalness as a guiding principle, a model explaining the 125GeV Higgs mass with the least fine tuning (so far as allowed by experimental bounds) would be favored.

- In this sense, Minimal Supersymmetric Standard Model (MSSM) is quite unsatisfactory.

  - The Higgs quartic term entirely comes from D-term, the gauge interaction.

$$m_{h\,\mathrm{tree}}^2 \leq m_Z^2 \cos^2 2\beta$$

This makes the tree level Higgs mass much smaller, need to be compensated by large quantum correction = heavy stop mass (several TeV)

- Moreover, MSSM is incomplete in a view of the basic spirit of solving hierarchy problem (mu problem):

The mu term in the MSSM superpotential $\mu H_u H_d$

introduces mass scale μ which is not in principle in the SUSY breaking scale, but it should be for successful electroweak symmetry breaking. We need an explanation.

- The singlet extension of suprsymmetric standard model is attractive as it can resolves these two problems. One famous example is the Next-to-Minimal Supersymmetric Standard Model(NMSSM)

The point is, we forbid the problematic mu term by introducing global U(1) symmetry, assigning to H_u and H_d the same charge, say, +1. Then the mu term is forbidden. Instead, we introduce a new singlet S, with U(1) charge -2. Then we have an alternate superpotential term

$$\lambda S H_u H_d$$

Then, all the mass scale in the potential is either the cutoff scale(say, Planck mass) or the SUSY breaking scale. If we can stabilize S with the VEV in the weak ~ SUSY breaking scale, then it would be a nice model. Mu problem solved

Moreover, the S direction F-term potential provides a new Higgs quartic term.

$$|\lambda H_u H_d|^2$$

This leads to a new tree level Higgs mass term

$$\delta m^2_{h\,\mathrm{tree}} = \lambda^2 v^2 \sin^2 2\beta$$

so we expect a enhanced tree level Higgs mass with order one lambda coupling.

Caution: lambda coupling tends to diverge at high energy. (Landau pole problem) So if we require perturbativity up to some scale, lambda cannot be large.

e.g. Perturbativity up to GUT scale requires $\lambda < 0.7$

If we hope to enhance a tree level Higgs mass with this new quartic term, large sin 2β, equivalently, small tan β is favored: different from MSSM, whose tree level Higgs mass favors large cos 2β =large tan β.

- Another question: What is use of this new U(1) symmetry?

Answer: If this U(1) symmetry is a vestige of U(1) symmetry broken at the intermediate scale, it can be used to explain extremely tiny CP violation in the strong interaction. We call this U(1) the Peccei-Quinn(PQ) symmetry.

When the strongly interacting particles (they may be either quarks we know or some high scale SU(3) particles) are charged under PQ symmetry either, the goldstone boson of PQ symmetry, axion obtains mass by QCD confinement and is stabilized such as to erase CP violating term

$$G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}$$

Through

$$
\begin{aligned}
e^{-\int d^4x V[a]} &= \left| \int \mathcal{D}A_\mu \prod_i \mathrm{Det}(\not{D} + m_i) e^{-\int d^4x \left[\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} - \frac{ia}{v_{\rm PQ}} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \right]} \right| \\
&\le \int \mathcal{D}A_\mu \left| \prod_i \mathrm{Det}(\not{D} + m_i) e^{-\int d^4x \left[\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} - \frac{ia}{v_{\rm PQ}} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \right]} \right| \\
&= \int \mathcal{D}A_\mu \prod_i \mathrm{Det}(\not{D} + m_i) e^{-\int d^4x \left[\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu} \right]} = e^{-\int d^4x V[0]}
\end{aligned}
$$

Why the PQ scale should be at the intermediate one?

This comes from observation.

The star cooling by axion renders the lower bound $v_{\rm PQ} > 10^9 \mathrm{GeV}$

The axion abundance by axionic dark matter imposes $v_{\rm PQ} < 10^{12} \mathrm{GeV}$

Then we have an interesting relation: $v_{\rm PQ} \sim \sqrt{m_{\rm SUSY} M_{\rm pl}}$

Can this be explained naturally? : A good mechanism for global(=accidental) PQ symmetry and its spontaneous breaking is required.

- Another request for a good SUSY model:

 - Flavor problem: Low energy new physics is helpful for hierarchy problem but potentially disasterous for flavor physics unless the squarks/sleptons are well aligned and (very close to) real to be safe from various flavor changing processes and CP violation bounds which are suppressed in the real world and the Standard Model describes them very well.

Model Building

- One way to describe the PQ symmetry breaking is just write down a superpotential

$$Z_1(S_1 S_2 - F_1^2)$$

It does not explain why PQ is broken in the intermediate scale.

Moreover, PQ is a global symmetry, broken by gravitational effects, so we need to explain a origin of the PQ symmetry

Instead, one can introduce a gauge symmetry, broken at much higher scale and regard the PQ as an vestige of the broken gauge symmetry.

- Consider an anomalous U(1) gauge symmetry, which frequently appears in the string model building.

It can be quantum mechanically consistent by Green-Schwarz anomaly cancellation mechanism:

When we have so called Green-Schwarz modulus T which transforms as

$$T \to T - \frac{\delta_{\mathrm{GS}}}{2}\Lambda$$

Under the U(1)_A gauge transformation

$$V_A \to V_A - \frac{1}{2}(\Lambda + \Lambda^*).$$

Where

$$\delta_{\mathrm{GS}} = \frac{1}{8\pi^2}\left(\frac{4}{3}\sum_i q_i^3\right) \sim \mathcal{O}\left(\frac{1}{8\pi^2}\right)$$

The Kaehler potential is a function of gauge invariant combination

$$t_A \equiv T + T^* - \delta_{\rm GS} V_A.$$

: GS modulus T can be regarded as a part of gauge trnsformation

This form implies that GS modulus T can be absorbed into gauge multiplet V. Then the vector multiplet V is massive. (Stueckelberg mechanism)

From Kaehler potential $$K = K_0(t_A, T_\alpha, T_\alpha^*) + \sum_i \phi_i^* e^{2q_i V_A} \phi_i,$$

With gauge transformation

$$\delta\Phi_I = \eta^I \Lambda,$$

$$\eta^T = -\frac{1}{2}\delta_{\rm GS}, \quad \eta^{T_\alpha} = 0, \quad \eta^{\phi_i} = q_i\phi_i.$$

We obtain $$M_A^2 = 2g_A^2 \eta^I \eta^J \partial_I \partial_{\bar J} K = 2g_A^2 (M_{\rm GS}^2 + \sum_i q_i^2 |\phi_i|^2)$$

$$D_A = -\eta^I \partial_I K = \xi_{\rm FI} - \sum_i q_i |\phi_i|^2$$

$$M_{\rm GS}^2 = \frac{\delta_{\rm GS}^2}{4} \partial_T \partial_{\bar T} K_0, \quad \xi_{\rm FI} = \frac{\delta_{\rm GS}}{2} \partial_T K_0$$

- In the supersymmetric vacuum, D_A =0:

We can achieve this by requiring

$$\xi_{\rm FI} = \sum_i q_i|\phi_i|^2 = 0$$

As matter VEV=0, gauge boson obtains mass by the Stueckelberg mechanism, rather than the Higgs mechanism.

- Now, suppose SUSY is broken by some modulus Z in the hidden sector.

The U(1)_A invariance implies that at stationary vacuum,

$$g_A^2 D_A = \frac{-g_A^2 F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L K_L) + V_D \eta^I \partial_I g_A^2}{V_F + 2|m_{3/2}|^2 + \frac{1}{2} M_A^2}. \simeq \frac{2}{\delta_{\rm GS}} \frac{\partial_t \partial_Z \partial_{\bar{Z}} K_0}{\partial_t^2 K_0} |F^Z|^2$$

$$\sim \left(\frac{\partial_t \partial_Z \partial_{\bar{Z}} K_0}{\partial_t^2 K_0 \partial_Z \partial_{\bar{Z}} K_0}\right) \frac{|m_{3/2}|^2}{\delta_{\rm GS}} \equiv \frac{\epsilon_1}{\delta_{\rm GS}} |m_{3/2}|^2.$$

For the first sequester-parameter $\epsilon_1 \sim 1/8\pi^2 \sim \delta_{\rm GS}$, we have $D_A \sim m_{3/2}^2$

$$m_i^2 = -q_i D_A.$$

- Backup slide

Now suppose that SUSY is mainly broken by some modulus Z, such that $\partial_Z \partial_{\bar{Z}} K_0 |F^z|^2 \simeq 3|m_{3/2}|^2$. In order to estimate D_A, one can use relations deduced in [11]. To begin with, recall that

$$
\begin{aligned}
&m_{3/2} = e^{K/2}W, \quad F^I = -e^{K/2}K^{I\bar{J}}(D_J W)^*, \quad D_I W = W_I + K_I W, \\
&V_F = K_{I\bar{J}}F^I F^{\bar{J}} - 3e^K|W|^2, \quad V_D = \frac{g_A^2}{2}D_A^2.
\end{aligned}
\tag{8}
$$

The $\mathrm{U}(1)_A$ invariance implies $\eta^I W_I = 0$ for superpotential and $\eta^I K_I = \eta^{\bar{I}} K_{\bar{I}}$ for Kähler potential, respectively. From this following relations can be obtained:

$$
\begin{aligned}
&m_{3/2}D_A = \eta^I F^{\bar{J}} K_{I\bar{J}}, \quad \eta^I D_I W = -W D_A, \\
&\eta^I \partial_I D_A = -\frac{M_A^2}{2g_A^2}, \quad \eta_L D_I W + \eta^I \partial_I (D_L W) = W \eta^{\bar{I}} K_{L\bar{I}}.
\end{aligned}
\tag{9}
$$

Then, for V_F and V_D, after careful calculations, relations

$$
\begin{aligned}
&\eta^I \partial_I V_F = -(V_F + 2|m_{3/2}|^2)D_A - F^I F^{\bar{J}} \partial_I (\eta^L K_{L\bar{J}}) \\
&\eta^I \partial_I V_D = V_D \eta^I \partial_I (\ln g_A^2) - \frac{1}{2}M_A^2 D_A
\end{aligned}
\tag{10}
$$

are satisfied and therefore we have

$$
(V_F + 2|m_{3/2}|^2 + \frac{1}{2}M_A^2)D_A = -F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L K_L) + V_D \eta^I \partial_I (\ln g_A^2) \tag{11}
$$

or, equivalently

$$
g_A^2 D_A = \frac{-g_A^2 F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L K_L) + V_D \eta^I \partial_I g_A^2}{V_F + 2|m_{3/2}|^2 + \frac{1}{2}M_A^2}. \tag{12}
$$

Since the SUSY is mainly broken by a modulus Z, a numerator is dominated by the first term $-g_A^2 F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L K_L) \simeq \frac{g_A^2}{2}\delta_{\mathrm{GS}} F^I F^{\bar{J}} K_{TI\bar{J}} \simeq \frac{g_A^2}{2}\delta_{\mathrm{GS}} |F^Z|^2 K_{TZ\bar{Z}}$. As a denominator is dominated by $\frac{1}{2}M_A^2 = g_A^2 \eta^I \eta^{\bar{J}} K_{I\bar{J}} \simeq \frac{1}{4}\delta_{\mathrm{GS}}^2 K_{T\bar{T}}$, we can estimate

- On the other hand,

$$A_{ijk} \sim F^Z \partial_Z \ln(e^{-K_0} \mathcal{Z}_i \mathcal{Z}_j \mathcal{Z}_k) \sim \Big(\frac{\partial_Z \ln(e^{-K_0/3} \mathcal{Z})}{\sqrt{\partial_Z \partial_{\bar{Z}} K_0}}\Big) m_{3/2} \equiv \epsilon_2 m_{3/2}$$

$$\frac{M_a}{g_a^2} \sim -\frac{1}{8\pi^2} \sum_i \mathrm{Tr}(T_a^2(\Phi_i)) F^Z \partial_Z \ln(e^{-K_0/3} \mathcal{Z}_i) \sim \frac{1}{8\pi^2} \epsilon_2 m_{3/2}.$$

- Assuming (almost) no-scale structure,

$$K = -3\log(Z + Z^*) \text{ and } \partial W/\partial Z = 0$$

F_C= $\frac{1}{3} K_i F^i + e^{K/2} W^*$ =0

: Anomaly mediation, or A-term of $W = \lambda^N C^3 \phi^N$ $\Delta V = \lambda_N (3-N) \phi^N (F^C/C)$.

negligible. ,

- From this setup, we obtain soft parameters

$$m_i^2 \sim -q_i(8\pi^2\epsilon_1)m_{3/2}^2$$

$$M_a \sim \frac{1}{8\pi^2}\epsilon_2 m_{3/2} \qquad A \sim \epsilon_2 m_{3/2}.$$

and below U(1)_A gauge boson mass scale, accidental global U(1) symmetry is obeyed by U(1)_A charged low energy degrees of freedom. We will interpret it as a Peccei-Quinn symmetry.

Next Question: How to break this PQ symmetry?

In D-term mediation, scalar soft mass can be tachyonic: we can use this to break PQ symmetry.

- Consider superpotential $$W = y\frac{XY^{n+2}}{M_*^n}.$$

For this, we assign PQ charges such that $q_X + (n+2)q_Y = 0.$ and consider a case where q_X<0 and q_Y>0. Then by $m_i^2 \sim -q_i(8\pi^2\epsilon_1)m_{3/2}^2$

Y is tachyonic, so breaks PQ symmetry and induces a X VEV:

$$\begin{aligned} V(X,Y) = \frac{|y|^2}{M_*^{2n}}|Y|^{2(n+2)} + \frac{|y|^2}{M_*^{2n}}(n+2)^2|X|^2|Y|^{2(n+1)} \\ + m_X^2|X|^2 + m_Y^2|Y|^2 + \left(yA\frac{XY^{n+2}}{M_*^n} + \text{h.c.}\right). \end{aligned}$$

$$Y = \frac{M_*^{\frac{n}{n+1}}|m_Y|^{\frac{1}{n+1}}}{(n+2)^{\frac{1}{2(n+1)}}|y|^{\frac{1}{n+1}}}$$

$$X = -\frac{y^*}{(n+2)^{\frac{(n+2)}{2(n+1)}}|y|^{\frac{n+2}{n+1}}}\frac{M_*^{\frac{n}{n+1}}A^*|m_Y|^{\frac{n+2}{n+1}}}{(n+2)|m_Y|^2+|m_X|^2}.$$

- As A-term is one-loop suppressed in the D-term mediation, X VEV would be suppressed compared to Y VEV.

$$\left|\frac{X}{Y}\right| = \frac{|A||m_Y|}{\sqrt{n+2}[(n+2)|m_Y|^2 + m_X^2]} \sim \frac{|A|}{\sqrt{D_A}} \sim \epsilon_2$$

- Y VEV would be interpreted as a PQ scale:

  If M_* is Planck scale,

$$v_{\rm PQ} \equiv \langle Y \rangle \sim |y|^{-\frac{1}{n+1}} m_{3/2}^{\frac{1}{n+1}} M_*^{\frac{n}{n+1}} = \begin{cases} |y|^{-1/2} 10^{10}\,{\rm GeV} & n=1 \\ |y|^{-1/3} 10^{12-13}\,{\rm GeV} & n=2 \\ |y|^{-1/4} 10^{14}\,{\rm GeV} & n=3 \end{cases}$$

  If M_* is GUT scale,

$$v_{\rm PQ} \sim \begin{cases} |y|^{-1/2} 10^{9}\,{\rm GeV} & n=1 \\ |y|^{-1/3} 10^{11-12}\,{\rm GeV} & n=2 \\ |y|^{-1/4} 10^{13}\,{\rm GeV} & n=3 \end{cases}$$

Regarding PQ scale bound from cosmology $10^9\,{\rm GeV} < v_{\rm PQ} < 10^{12}\,{\rm GeV}$
n=1, 2 is favored.

- Moreover, we also have enhanced F_X/X.

$$\left|\frac{F_X}{X}\right| = \frac{(n+2)|m_Y|^2 + m_X^2}{|A|} \sim \frac{D_A}{A} \sim \frac{1}{\epsilon_2} m_{3/2}$$
$$\left|\frac{F_Y}{Y}\right| = \frac{|A||m_Y|^2}{(n+2)|m_Y|^2 + m_X^2} \sim A \sim \epsilon_2 m_{3/2}.$$

In short, $X \sim v_{\mathrm{PQ}}(\epsilon_2 + m_{3/2}\theta^2)$ while $Y \sim v_{\mathrm{PQ}}(1 + \epsilon_2 m_{3/2}\theta^2)$.

Actually, if D-term mediation is the only source of SUSY breaking in the SSM sector, gaugino masses are too suppressed: From gluino mass bund > 1.5TeV, squark/slepton can be too large.

Moreover, there may be cumbersome tachyonic squark/slepton/Higgs.

In this sense, we need more source for SUSY breaking in the visible sector.

Suppose X couples to some vector-like quark as $W = X\overline{Q}_1 Q_1$

If this pair is also charged under U(1)_Y and SU(2), it can be used as a messenger for gauge mediation, such that

$$M_{\mathrm{GMSB}} \equiv \frac{1}{8\pi^2}\left|\frac{F_X}{X}\right| \sim \frac{1}{8\pi^2}\frac{D_A}{A} \sim \frac{1}{8\pi^2\epsilon_2}\sqrt{D_A}.$$

- By making epsilon_2 one loop suppressed, gauge mediation effect can be comparable to D-term mediation effect, so it gives reasonable spectrum in a view of naturalness.

Higgs Sector with additional Singlet(s)

- Goal: For successful electroweak symmetry breaking (EWSB), we need mu and Bmu term in an appropriate size, neither too large nor too small, without mu-problem.
- For this, we introduce a SM singlet S with PQ charge such that we have a low energy superpotential

$$W = \lambda S H_u H_d + f(S)$$

- Then we have

$$\mu_{\text{eff}} = \lambda \langle S \rangle, \qquad (B\mu)_{\text{eff}} = \lambda (A_\lambda \langle S \rangle + \langle \partial_S f \rangle)$$

- So, mu and Bmu parameters are determined by coefficients of singlet superpotential, which will be explain as VEVs or F-term VEVs of PQ sector fields, X and Y.

- We can consider generic form of superpotential,

$$W = \xi(1+\theta^2 C)S + \frac{1}{2}\mu'(1+\theta^2 B')S^2 + \lambda(1+\theta^2 A_\lambda)SH_uH_d.$$

As we impose PQ symmetry, rather than Z_3 symmetry(used in conventional NMSSM), dimensionless coupling to S^3 term is forbidden at tree level and suppressed as (v_PQ/M_*)^n even if S couples to X or Y. So we do not consider it.

- For successful EWSB, parameters should satisfy

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} - \mu_{\rm eff}^2$$

$$\sin 2\beta = \frac{2(\lambda\xi + \mu'\mu_{\rm eff} + A_\lambda\mu_{\rm eff})}{2\mu_{\rm eff}^2 + m_{H_u}^2 + m_{H_d}^2 + \lambda^2 v^2}$$

$$\mu_{\rm eff} = \frac{-\xi\mu' - C\xi + \lambda(A_\lambda + \mu')v_u v_d}{m_S^2 + \lambda^2 v^2 + \mu'^2 + B'\mu'}$$

For EWSB with least fine-tuning (sin2beta ~ O(1)) in our setup,

$$m_S^2 \sim D_A, \qquad m_{H_{u,d}}^2 \sim D_A + M_{\mathrm{GMSB}} \sim D_A$$

What do we need?

- We can find that B’mu’ term play the same role as soft mass m_S^2, it is sufficient for it not to exceed this soft mass scale. (It is OK even it is too suppressed)

- On the other hand, at least two of other three parameters should be of order of D_A. This can be implemented by following way:

  keeping our relation $v_{\mathrm{PQ}}^{n+1} = M_{\mathrm{pl}}^n \sqrt{D_A}$ into mind,

$$X \sim v_{\mathrm{PQ}}(\epsilon_2 + m_{3/2}\theta^2) \text{ while } Y \sim v_{\mathrm{PQ}}(1 + \epsilon_2 m_{3/2}\theta^2).$$

1. μ' is obtained from a superpotential term

$$\Delta W = \frac{Y^{n+1}}{M_*^n} S^2.$$

For this, X and Y should have PQ charges

$$q_X = \frac{2(n+2)}{n+1} q_S, \quad q_Y = -\frac{2}{n+1} q_S$$

respectively. For D-term soft mass m_Y^2 to be tachyonic, $q_Y > 0$ so $q_S < 0$.

2. ξ is obtained from a Kähler potential term

$$\Delta K = \frac{X^* Y^{*n}}{M_*^n} S$$

by taking F_X^* and Y VEVs, which requires

$$q_X = \frac{n+2}{2} q_S, \quad q_Y = -\frac{1}{2} q_S$$

or a Kähler- and a superpotential term

$$\Delta K = \frac{X^* Y^n}{M_*^n} S, \qquad \Delta W = \frac{Y^{2n+2}}{M_*^{2n}} S$$

with

$$q_X = \frac{n+2}{2(n+1)} q_S, \quad q_Y = -\frac{1}{2(n+1)} q_S.$$

3. $C\xi$ is obtained from a F-term of superpotential term

$$\Delta W = \frac{XY^{2n+1}}{M_*^{2n}} S$$

by taking F_X and Y VEVs, with

$$q_X = \frac{n+2}{n-1} q_S, \quad q_Y = -\frac{1}{n-1} q_S.$$

Such charge assignments show that any two of three coincide with each other for the case of our interest, n=1, 2.

(In fact, we may find coincident for n=3, but in this case, large tadpole from W=Y^2S appears, and it cannot be forbidden by introducing additional symmetry)

That means, if we set any of three at desired scale, other parameters are too large or too small.

This is, in fact, simply because our setup is too simple: parameters we need is provided from Y VEV and F_X VEV only as X VEV and F_Y VEV are suppressed.

We have two solution here:

- Consider the third PQ sector field, say Z, which can divide the role of Y VEV or F_X VEV.

Example:

$$W = y\frac{XY^{n+1}Z^{m+1}}{M_*^{n+m}},$$

with n, m>1, then $Y \sim Z \equiv v_{\rm PQ}$ while X VEV is still suppressed by A-term.

In low energy point of view, Y and Z share its role, but have different PQ charges. Then some of parameters are coming from Y instead of Z and we have a chance that at least two parameters are in desired scale, and others are suppressed or at the same order.

For $W = y\frac{XY^2Z^2}{M_*^2}$ we have following possible models:

<table>
<tr><th>ξ</th><th>$C\xi$</th><th>μ'</th><th>(r,s)</th><th>(q_X, q_Y, q_Z)</th><th>Dangerous terms</th></tr>
<tr><td rowspan="4">$\Delta W = \frac{Y^r Z^{6-r}}{M_*^4} S$</td><td rowspan="4">$\Delta W = \frac{XY^s Z^{5-s}}{M_*^4} S$</td><td rowspan="4"></td><td>$(0,4)$</td><td>$(\frac{3}{2}, -\frac{7}{12}, -\frac{1}{6}) q_S$</td><td></td></tr>
<tr><td>$(0,5)$</td><td>$(\frac{11}{9}, -\frac{4}{9}, -\frac{1}{6}) q_S$</td><td></td></tr>
<tr><td>$(1,4)$</td><td>$(\frac{14}{11}, -\frac{6}{11}, -\frac{1}{11}) q_S$</td><td></td></tr>
<tr><td>$(1,5)$</td><td>$(\frac{18}{17}, -\frac{7}{17}, -\frac{2}{17}) q_S$</td><td></td></tr>
<tr><td>$\Delta W = \frac{Y^2 Z^4}{M_*^4} S,$
$\Delta K = \frac{X^* Z^2}{M_*^2} S$</td><td>$\Delta W = \frac{XY^5}{M_*^4} S$</td><td></td><td></td><td>$(\frac{7}{8}, -\frac{3}{8}, -\frac{1}{16}) q_S$</td><td></td></tr>
<tr><td rowspan="4">$\Delta W = \frac{Y^r Z^{6-r}}{M_*^4} S$</td><td rowspan="4"></td><td rowspan="4">$\Delta W = \frac{Y^s Z^{3-s}}{M_*^2} S^2$</td><td>$(0,1)$</td><td>$(\frac{11}{3}, -\frac{5}{3}, -\frac{1}{6}) q_S$</td><td></td></tr>
<tr><td>$(5,0)$</td><td>$(\frac{22}{15}, -\frac{1}{15}, -\frac{2}{3}) q_S$</td><td></td></tr>
<tr><td>$(6,0)$</td><td>$(\frac{5}{3}, -\frac{1}{6}, -\frac{2}{3}) q_S$</td><td>$\Delta K = \frac{X^* Z^*}{M_*} S,$
$\Delta W = \frac{Y^2 Z}{M_*} S + \frac{XZ^4}{M_*^3} S$</td></tr>
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<tr><td rowspan="5"></td><td rowspan="5">$\Delta W = \frac{XY^r Z^{5-r}}{M_*^4} S$</td><td rowspan="5">$\Delta W = \frac{Y^s Z^{3-s}}{M_*^2} S^2$</td><td>$(0,1)$</td><td>$(\frac{18}{7}, -\frac{4}{7}, -\frac{5}{7}) q_S$</td><td></td></tr>
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<tr><td>$(4,1)$</td><td>$(\frac{14}{5}, -\frac{4}{5}, -\frac{3}{5}) q_S$</td><td></td></tr>
<tr><td>$(5,0)$</td><td>$(\frac{26}{9}, -\frac{7}{9}, -\frac{2}{3}) q_S$</td><td></td></tr>
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</table>

Z_3 symmetry with $X, Z = 1$ and $Y, S = 0$.

$$W_{\mathrm{I}} = \xi(1 + C\theta^2)S + \lambda S H_u H_d$$

$$W_{\mathrm{II}} = \xi S + \frac{1}{2}\mu' S^2 + \lambda S H_u H_d$$

$$W_{\mathrm{III}} = \xi(C\theta^2)S + \frac{1}{2}\mu' S^2 + \lambda S H_u H_d$$

$$m_{h_{\mathrm{tree}}}^2 = \lambda^2 v^2 \sin^2 2\beta \frac{m_S^2}{\mu'^2 + m_S^2}.$$

- Another solution: parameters of singlet S are provided by not just coupling to X, Y but also coupling to another singlet.

Dynamics of another singlet is regulated by parameters which will be determined by its coupling to X, Y.

In general, we can think of

$$W = \lambda S_1 H_u H_d + \frac{1}{2}\kappa_1(1+\theta^2 A_1)S_1^2 S_2 + \kappa_2 \frac{1}{2}(1+\theta^2 A_2)S_1 S_2^2$$
$$+ \frac{1}{2}\mu_1(1+\theta^2 B_1)S_2 + \frac{1}{2}\mu_2(1+\theta^2 B_2)S_2^2 + M(1+\theta^2 B_M)S_1 S_2$$
$$+ \xi_1(1+\theta^2 C_1)S_1 + \xi_2(1+\theta^2 C_2)S_2.$$

But not so long as S_1 and S_2 have the same PQ charge, only one of corresponding terms is allowed.

1. In the presence of cubic term

$$W = \frac{1}{2}\kappa_1 S_1^2 S_2 + \xi_1 S_1 + \lambda S_1 H_u H_d.$$

we can make quartic term of S_1 in the potential, so S_1 is stabilized. If soft mass of S_1 tachyonic, S_1 can have VEV, which provides mu term.

On the other hand Bmu term comes from \xi_1 or S_2 VEV induced from S_2 tadpole (<- cubic term)

S_2 should not be tachyonic, as potential is at most quadratic in S_2 so we cannot stabilize it. (Even though it is stabilized by largely suppressed non-renormalizable term, it implies large S_2 VEV so too large Bmu term appears)

in our model,

The ξ_1 comes from a VEV of Y^{2n+2}/M_*^{2n} or F-term VEV of X^*Y^{*n}/M_*^n or X^*Y^n/M_*^n

but this implies S_2, instead of S_2 is tachyonic since Y and S_1 have PQ charges in opposite sign.

So we need to consider

$$W = \frac{1}{2}\kappa_1 S_1^2 S_2 + \xi_2 S_2 + \lambda S_1 H_u H_d$$

to make S_1 tachyonic while S_2 not. But this is also unacceptable as S_2 is stabilized at origin so canot generate Bmu term.

For S_2 to get VEV,

$$W = \frac{1}{2}\kappa_1 S_1^2 S_2 + \theta^2 C \xi_2 S_2 + \lambda S_1 H_u H_d$$

is fine.

Unfortunately, this is not realized in our model as charge assignment for it is

$$q_X = -2\frac{n+2}{n-1} q_{S_1}, \quad q_Y = \frac{2}{n-1} q_{S_1}, \quad q_{S_2} = -2 q_{S_1}.$$

Such that the solution does not exist for n=1 and for n=2, too large tadpole from YS_2 is also allowed.

For

$$W = \frac{1}{2}\kappa_1 S_1^2 S_2 + M S_1 S_2 + \lambda S_1 H_u H_d.$$

M comes from VEV of Y^{n+1}/M_*^n, we need

$$q_X = -\frac{n+2}{n+1} q_{S_1}, \quad q_Y = \frac{1}{n+1} q_{S_1}, \quad q_{S_2} = -2 q_{S_1}.$$

This allows tadpole $\xi_2 S_2$ from $Y^{2(n+1)} S_2 / M_*$ so both S_1 and S_2 have VEVs. But…. We also have dangerous tadpole XYS_1 either!

In this way, we find

$$W = \frac{1}{2}\kappa_2 S_2^2 S_1 + M(B_M \theta^2) S_1 S_2 + \lambda S_1 H_u H_d$$

also cannot be realized by large tadpole term.

Actually, large tadpole may be avoided of we introduce additional PQ sector field Z etc.

- In the end, the viable model with cubic term is

$$W_{\mathrm{IV}} = \frac{1}{2}\kappa_1 S_1^2 S_2 + \frac{1}{2}\mu_2 S_2^2 + \lambda S_1 H_u H_d.$$

This is viable in the sense of charge assignment. A μ_2 term can be generated from a VEV of Y^{n+1}/M_*^n coupling to S_2^2 in the superpotential. Then we have a charge assignment

$$q_X = -\frac{4(n+2)}{n+1} q_{S_1}, \quad q_Y = \frac{4}{n+1} q_{S_1}, \quad q_{S_2} = -2q_{S_1}.$$

For $n = 1$, we have a dangerous tadpole term YS_2 but this can be forbidden by introducing ng Z_2 symmetry under which X and Y are odd while $S_{1,2}$ are even. Then we can have a full superpotential

$$W = y\frac{XY^3}{M_*} + \frac{1}{2}y'\frac{Y^2}{M_*}S_2^2 + \frac{1}{2}\kappa_1 S_1^2 S_2$$

without dangerous tadpole terms. On the other hand, for $n = 2$, we have

$$W = y\frac{XY^4}{M_*^2} + \frac{1}{2}y'\frac{Y^3}{M_*}S_2^2 + \frac{1}{2}\kappa_1 S_1^2 S_2$$

$$\mu_{\text{eff}} = \lambda\langle S_1\rangle \sim \frac{m_{S_1}}{\kappa_1} \sim \frac{\sqrt{D_A}}{\kappa_1}$$

$$(B\mu)_{\text{eff}} = \lambda\kappa_1\langle S_1 S_2\rangle \sim \frac{m_{S_1}\mu_2}{\kappa_1} \sim \frac{y'}{\kappa_1}D_A.$$

On the other hand, for

$$W_{\rm V} = \frac{1}{2}\kappa_2 S_2^2 S_1 + \frac{1}{2}\mu_1 S_1^2 + \lambda S_1 H_u H_d,$$

as $\langle S_1\rangle \sim \mu_1/\kappa_2$ and $\langle S_2\rangle \sim |m_{S_2}|/\kappa_2$, we have

$$\begin{aligned} \mu_{\rm eff} &\sim \lambda\langle S_1\rangle \sim \frac{\mu_1}{\kappa_2} \sim \frac{y'}{\kappa_2}\sqrt{D_A} \\ (B\mu)_{\rm eff} &= \lambda\Big\langle \frac{1}{2}\kappa_2 S_2^2 + \mu_1 S_1\Big\rangle \sim \frac{m_{S_2}^2}{\kappa_2} \sim \frac{D_A}{\kappa_2}. \end{aligned}$$

For $\mu^2 \sim B\mu$, we require $\kappa_2 \sim y'^2$. It provides interesting possibility that, the EWSB can be successful even when $\sqrt{D_A}$ is smaller than the gauge mediation contribution, which will be reflected in $m_{H_{u,d}}^2$: from $\mu^2 \sim B\mu \sim m_{H_u}^2$, we should have $\sqrt{D_A} \sim y'|m_{H_u}|$ and by definition $\mu_1 \sim y'\sqrt{D_A} \sim y'^2 m_{H_u}^2$ so for $y' < 1$, we have a hierarchy $m_{H_u} > \sqrt{D_A} > \mu_1$. Such hierarchy in mass scale implies the existence of several low mass degrees of freedom.

- Case without cubic term:

$$W = M(1+\theta^2 B_M)S_1 S_2 + \xi_2(1+\theta^2 C_2)S_2 + \lambda S_1 H_u H_d,$$

in which singlet VEVs are given by

$$\langle S_1\rangle = \frac{-(m_{S_2}^2+M^2)M\xi_2 + (B_M M)(C_2\xi_2)}{(m_{S_1}^2+M^2)(m_{S_2}^2+M^2)-(B_M M)^2}$$
$$\langle S_2\rangle = \frac{-(m_{S_2}^2+M^2)(C_2\xi_2) + (B_M M)M\xi_2}{(m_{S_1}^2+M^2)(m_{S_2}^2+M^2)-(B_M M)^2}.$$

From $\mu = \lambda\langle S_1\rangle$ and $B\mu = M\langle S_2\rangle + \xi_2$, we need to consider two cases: $(M \sim \sqrt{D_A}, \xi_2 \sim D_A)$ and $(B_M M \sim D_A, C_2\xi_2 \sim D_A^{3/2})$.

1. $(M \sim \sqrt{D_A}, \xi_2 \sim D_A)$

This case,

$$W = MS_1S_2 + \xi_2 S_2 + \lambda S_1 H_u H_d$$

This is interesting as we can enhance the tree level Higgs mass without requiring large m_{S_1}^2 , by replacing it by m_{S_2}^2. So at least, we can make small fine tuning for the Higgs.

$$m_{h_{\rm tree}}^2 = \lambda^2 v^2 \sin^2 2\beta \frac{m_{S_2}^2}{M^2 + m_{S_2}^2}$$

2. $(B_M M \sim D_A, C_2\xi_2 \sim D_A^{3/2})$

Next, consider a suppressed M, but large $B_M M \sim D_A$ and $C_2\xi_2 \sim D_A^{3/2}$ through

$$W = M(B_M\theta^2)S_1S_2 + \xi_2(C_2\theta^2)S_2.$$

But these two cases are not realized in our setup either as large tadpole term is allowed.

This may be resolved in another setup, for example three PQ sector fields.

Conclusion

- Even though LHC have not put forward evidence of new physics, new physics is worth to explore due to incompleteness of the Standard Model in both theoretical and phenomenological view.
- If you still consider naturalness as a guiding principle of new physics and if you still hope that new physics can appear near future, supersymmetry is quite a good option.
- Especially, singlet extension is worth to study, especially related to the Peccei-Quinn symmetry, physics of intermediate scale.
- Intermediate scale can be explained by interplay between SUSY breaking and cut-off suppressed non-renormalizable interaction.
- There are several types of models for singlet extended Higgs sector which can be realized by non-renormalizable interaction between singlet and PQ sector.
- Some of them are phenomenologically interesting.