

## Searching for composite quark partners at the LHC



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C. Delaunay, TF, J. Gonzales-Fraile,  
S.J. Lee, G. Panico, G. Perez [JHEP 02 (2014) 055]

TF, Jeong Han Kim,  
Seung Joon Lee, Sung Hak Lim [arXiv:1312.5316]

TF, Sang Eun Han,  
Jeong Han Kim, Seung Joon Lee *in preparation*

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# Outline

- Motivation
- The general setup: minimal composite Higgs from  $SO(5)/SO(4)$  breaking
- Partially composite quarks
  - The Lagrangian
  - Partners in the  $SO(4)$  fourplet
  - Partners in the  $SO(4)$  singlet
  - Singlet and fourplet partners: how generic are pure fourplet constraints?
- Conclusions and Outlook
- Backup: Fully composite quarks

# Motivation

- ☺ Atlas and CMS found a Higgs-like resonance with a mass  $m_h \sim 126$  GeV and couplings to  $\gamma\gamma$ ,  $WW$ ,  $ZZ$ ,  $bb$ , and  $\tau\tau$  compatible with the standard model Higgs.
- ☹ The standard model suffers from the hierarchy problem.

⇒ We need to search for an SM extension with a Higgs-like state which provides an explanation for why  $m_h, v \ll M_{pl}$ .

One possible solution: Composite Higgs Models (CHM)

- Consider a model which gets strongly coupled at a scale  $f \sim \mathcal{O}(1 \text{ TeV})$ .  
→ naturally obtain  $f \lll M_{pl}$ .
- Assume a global symmetry which is spont. broken by dim. transmutation.  
→ strongly coupled resonances at  $f$   
and Goldstone bosons (to be identified with the Higgs sector).
- Assume that the only source of explicit symmetry breaking arises from Yukawa-type interactions.  
→ The Higgs-like particles become pseudo-Goldstone bosons  
⇒ Naturally generates a scale hierarchy  $v \sim m_h \ll f \lll M_{pl}$ .

## Composite Higgs model: general setup

### Simplest realization:

The minimal composite Higgs model (MCHM) Agashe, Contino, Pomarol [2004]

Effective field theory based on  $SO(5) \rightarrow SO(4)$  global symmetry breaking.

- The Goldstone bosons live in  $SO(5)/SO(4) \rightarrow 4$  d.o.f.
- $SO(4) \simeq SU(2)_L \times SU(2)_R$

Gauging  $SU(2)_L$  yields an  $SU(2)_L$  Goldstone doublet.

Gauging  $T_R^3$  assigns hyper charge to it. Later: Include a global  $U(1)_X$  and gauge  $Y = T_R^3 + X$ .

$\Rightarrow$  Correct quantum numbers for the Goldstone bosons to be identified as a non-linear realization of the Higgs doublet.

We use the CCWZ construction to construct the low-energy EFT.

Coleman, Wess, Zumino [1969], Callan, Coleman [1969]

Central element: the Goldstone boson matrix

$$U(\Pi) = \exp\left(\frac{i}{f}\Pi_i T^i\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \bar{h}/f & \sin \bar{h}/f \\ 0 & 0 & 0 & -\sin \bar{h}/f & \cos \bar{h}/f \end{pmatrix},$$

where  $\Pi = (0, 0, 0, \bar{h})$  with  $\bar{h} = \langle h \rangle + h$   
 and  $T^i$  are the broken  $SO(5)$  generators.

From it, one can construct the CCWZ  $d_\mu^j$  and  $e_\mu^a$  symbols (roughly speaking: connections corresponding to broken / unbroken generators).  
E. g. kinetic term for the “Higgs”:

$$\mathcal{L}_\Pi = \frac{f^2}{4} d_\mu^j d^{j\mu} = \frac{1}{2} (\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2 \left( \frac{\bar{h}}{f} \right) \left( W_\mu W^\mu + \frac{1}{2c_w} Z_\mu Z^\mu \right)$$

$$\Rightarrow v = 246 \text{ GeV} = f \sin \left( \frac{\langle h \rangle}{f} \right) \equiv f \sin(\epsilon).$$

**Note:** In the above, the Higgs multiplet is parameterized as a Goldstone multiplet and it is *assumed* that a Higgs potential is induced which leads to EWSB.

Concrete realizations *c.f. e.g. Review by Contino [2010], Panico et al. [2012], ...*:

Couplings of the Higgs to the quark sector (most importantly to the top) explicitly break the  $SO(5)$  symmetry

$\Rightarrow$  couplings to the top sector induce an effective potential for the Higgs which induces EWSB.

## How to include the quarks?

In the SM, the Higgs multiplet

- induces EWSB (✓ in CHM),
- provides a scalar degree of freedom (✓ in CHM),
- generates fermion masses via Yukawa terms (← implementation in CHM?).

**One solution** [Kaplan (1991)]: Include elementary fermions  $q$  as incomplete linear reps of  $SO(5)$  which couple to the strong sector via

$$\mathcal{L}_{mix} = y \bar{q}_{l_0} \mathcal{O}^{l_0} + \text{h.c.}$$

where  $\mathcal{O}$  is an operator of the strongly coupled theory in the rep.  $l_0$ .

**Note:** The Goldstone matrix  $U(\Pi)$  non-linearly under  $SO(5)$ , but linear under the  $SO(4)$  subgroup  $\rightarrow \mathcal{O}^{l_0}$  has the form  $f(U(\Pi))\mathcal{O}'_{fermion}$ .

Simplest choice for quark embedding:

$$q_L^5 = \frac{1}{\sqrt{2}} \begin{pmatrix} id_L \\ d_L \\ iu_L \\ -u_L \\ 0 \end{pmatrix}, \quad u_R^5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_R \end{pmatrix}, \quad \psi = \begin{pmatrix} Q \\ \tilde{U} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} iD - iX_{5/3} \\ D + X_{5/3} \\ iU + iX_{2/3} \\ -U + X_{2/3} \\ \sqrt{2}\tilde{U} \end{pmatrix}.$$

# How to include the quarks?

## Remarks:

- Another “as minimal” embedding as considered here:  
embed  $q_L$  and  $\psi$  in the same way in the **5** and  $u_R$  as a chiral  $SO(5)$  singlet.  
⇒ “(fully) composite right-handed quarks *c.f. e.g. Rattazzi et al. [2012]*  
(We studied this second case in detail, too. → backup slides)
- The choice of rep. for the light quarks and their partners is not unique.  
Other embeddings which are sometimes discussed:
  - **14** = **1**  $\oplus$  **4**  $\oplus$  **9** *c.f. e.g. Rattazzi et al. [2012], Panico et al. [2012], Torre et al. [2013]*  
One qualitative new feature:  
The **9** contains additional partner particles with exotic charges.
  - **10** = **4**  $\oplus$  **6** *c.f. e.g. Contino et al. [2006], Redi et al. [2008], Panico et al. [2010], Azatov et al. [2011], Mühlleitner et al. [2013]*  
New features:  
New partners in the **6**.  
Quark partners in the **10** can serve as up- *and* down-type partners simultaneously.

Back to partially composite quarks in the **5**.  
 BSM particle content:

	$U$	$X_{2/3}$	$D$	$X_{5/3}$	$\tilde{U}$
$SO(4)$	<b>4</b>	<b>4</b>	<b>4</b>	<b>4</b>	<b>1</b>
$SU(3)_c$	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>
$U(1)_X$ charge	2/3	2/3	2/3	2/3	2/3
EM charge	2/3	2/3	-1/3	5/3	2/3

Fermion Lagrangian:

$$\mathcal{L}_{comp} = i \bar{Q}(D_\mu + ie_\mu)\gamma^\mu Q + i \bar{U} \not{D} \tilde{U} - M_4 \bar{Q} Q - M_1 \bar{U} \tilde{U} + (i c \bar{Q}^i \gamma^\mu d_\mu^j \tilde{U} + \text{h.c.}),$$

$$\mathcal{L}_{el,mix} = i \bar{q}_L \not{D} q_L + i \bar{u}_R \not{D} u_R - y_L f \bar{q}_L^5 U_{gs} \psi_R - y_R f \bar{u}_R^5 U_{gs} \psi_L + \text{h.c.},$$

Derivation of Feynman rules:

- expand  $d_\mu$ ,  $e_\mu$ ,  $U_{gS}$  around  $\langle h \rangle$ ,
- diagonalize the mass matrices,
- match the lightest up-type mass with the SM quark mass  
→ this fixes  $y_L$  in terms of the other parameters ( $y_R \sim 1 \Rightarrow y_L \ll 1$ )
- calculate the couplings in the mass eigenbasis.

## Partners in the fourplet

Lets first consider the limit  $M_1 \rightarrow \infty$ .

$\tilde{U}$  decouples, and the remaining quark partners form a **4** of  $SO(4)$ .

Mass eigenstates:

$$U_{p/m} = (1/\sqrt{2}) (U \pm X_{2/3}), D, X_{5/3}.$$

Masses:

$$m_{U_p} = m_D = m_{X_{5/3}} = M_4, m_{U_m} = \sqrt{M_4^2 + (y_R f \sin(\epsilon))^2}, \text{ with } \epsilon = \langle h \rangle / f.$$

“Mixing” couplings:

$$\begin{aligned} g_{WuX} = -g_{WuD} = -c_w g_{ZuU_p} &= \frac{g}{2} \cos \epsilon \sin \varphi_4, \\ \lambda_{huU_m} &= y_R \cos \epsilon \cos \varphi_4, \end{aligned}$$

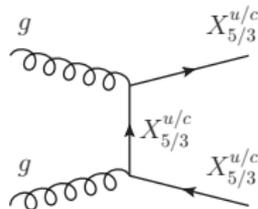
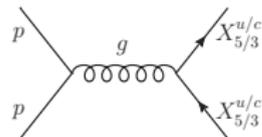
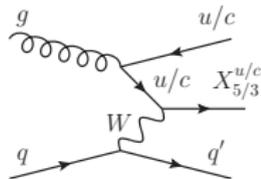
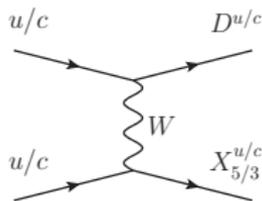
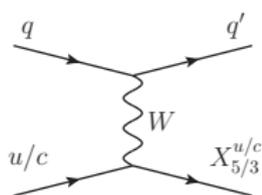
with

$$\tan \varphi_4 \equiv \frac{y_R f \sin \epsilon}{M_4}.$$



# Partners in the fourplet

Production mechanisms (shown here:  $X_{5/3}^{u/c}$  production)



(a) EW single production

(b) EW pair production

(c) QCD pair production

Decays:

- $X_{5/3} \rightarrow W^+ u$  (100%),
- $D \rightarrow W^- u$  (100%),
- $U_p \rightarrow Zu$  (100%),
- $U_m \rightarrow hu$  (100%).

## NOTE:

- The EW production mechanisms strongly differs for 1st, 2nd, and 3rd generation partners due to the differing PDFs for  $u, c, t$  in the proton.
- The final states (search signatures) differ:
  - 1st generation partners:  $u, d$  quarks in the final state  $\rightarrow$  jets.
  - 2nd generation partners:  $c, s \rightarrow$  jets, potentially tagable  $c$  in the future
  - 3rd generation partners:  $t, b \rightarrow$  well distinguishable from jets

We focus on 1st and 2nd family partners.

c.f. [Rattazzi *et al.* (2012)] for top partners.

c.f. [TF, S.E. Han, J.H. Kim, S.J. Lee, (to appear soon)] for bottom partners.

$\rightarrow$  relevant measured final states:

- Single production:  $Wjj, Zjj$

[D0 Collaboration], Phys. Rev. Lett. 106, 081801 (2011)

[CDF Collaboration], CDF/PUB/EXOTIC/PUBLIC/1026

[ATLAS Collaboration], ATLAS-CONF-2012-137 (4.64  $fb^{-1}$  7 TeV)

[CMS Collaboration], CMS-PAS-EXO-12-024 (19.8  $fb^{-1}$  8 TeV)

- Pair production:  $WWjj, ZZjj$

[D0 Collaboration], Phys. Rev. Lett. 107, 082001 (2011)

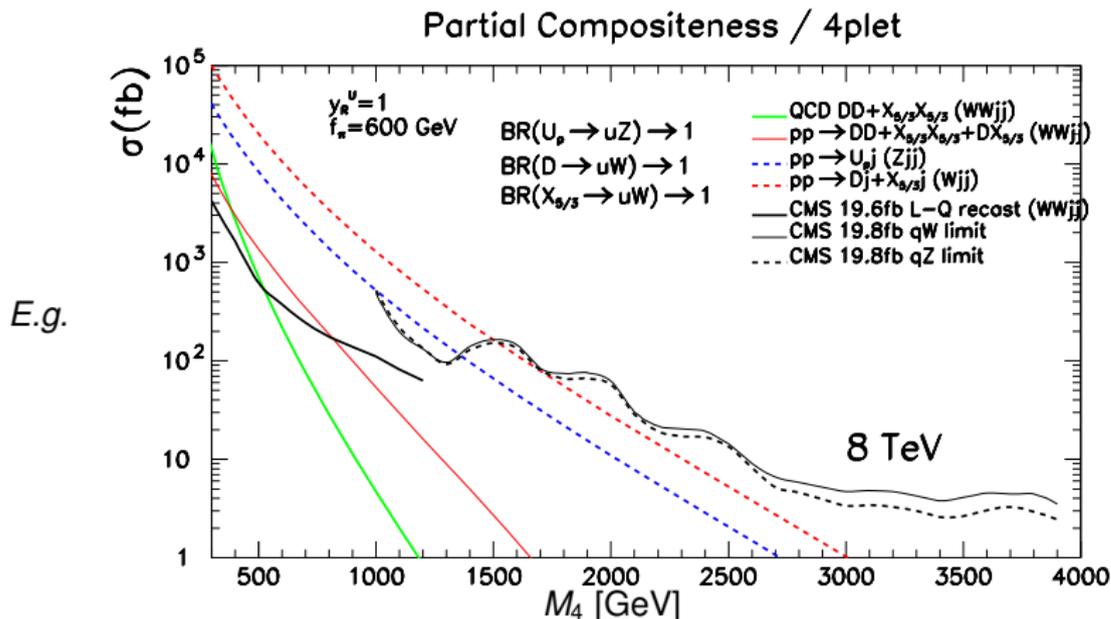
[CDF Collaboration], Phys. Rev. Lett. 107, 261801 (2011)

[ATLAS Collaboration], Phys. Rev. D 86, 012007 (2012) (1.04  $fb^{-1}$  7 TeV)

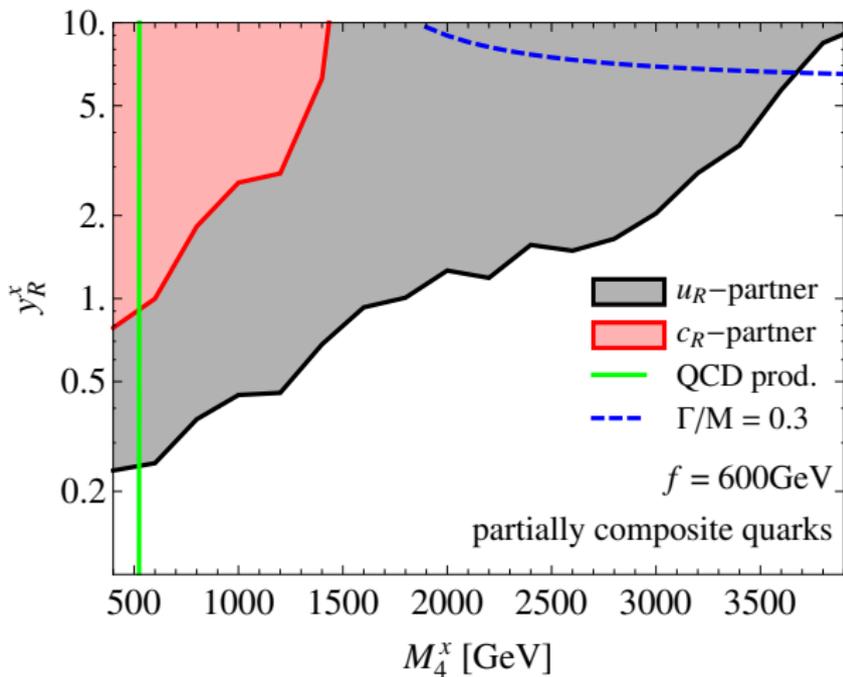
[CMS Collaboration], CMS-PAS-EXO-12-042 (19.6  $fb^{-1}$  8 TeV); Leptoquark search, final state:  $\mu\mu jj$

## Determining bounds from searches

- To determine the bounds from Tevatron, ATLAS and CMS searches we
- implement the model [FeynRules2.0 → MadGraph5 (using CTEQ6L)],
  - simulate the BSM signals on parton level,
  - compare with the bounds established by the experimental searches.



# Determining bounds from searches



[JHEP 02 (2014) 055]. Analysis for bottom partners is under way

## Partners in the singlet (qualitative discussion)

Now lets look at the opposite limit:  $M_1$  finite and  $M_4 \rightarrow \infty$ .

Then, all fourplet states decouple, and the only remaining BSM state is  $\tilde{U}$ .

$$\text{Mass: } m_{\tilde{U}} = \sqrt{M_1^2 + (y_R f \cos(\epsilon))^2}$$

only “mixing” coupling:

$$\lambda_{hu\tilde{U}} = y_R \sin \epsilon \cos \varphi_1, \quad \text{with} \quad \tan \varphi_1 \equiv \frac{y_R f \cos \epsilon}{M_1}.$$

Production: pair-production (QCD and EW)

Decay:  $\tilde{U} \rightarrow hj$  (100%)

Signal:  $pp \rightarrow hhjj$ .

No data on the di-higgs channel was available at the time of our study.

$\Rightarrow$  Only “theory” bound:  $m_{\tilde{U}} > m_h$  (otherwise Higgs BR are modified).

## General case: $M_1$ and $M_4$ finite.

We have obtained bounds on the fourplet partners with the singlet decoupled.

How are these bounds modified when the singlet is not decoupled?

BSM Particle content:  $X_{5/3}, D, U_p, U_1, U_2$

Where  $U_{1,2}$  are the mass eigenstates of  $U_m - \tilde{U}$  mixing.

Masses:  $m_{U_p} = m_D = m_{X_{5/3}} = M_4, m_{U_{1,2}} =$

$$\frac{1}{2} \left[ M_1^2 + M_4^2 + y_R^2 f^2 \mp \sqrt{(M_1^2 - M_4^2 + y_R^2 f^2)^2 - 4 \sin^2 \epsilon (M_1^2 - M_4^2) y_R^2 f^2} \right].$$

“mixing” couplings with light quarks:

$$\begin{aligned} \lambda_{hU_1} &\approx -y_R \cos \epsilon \cos \varphi_4 \cos \tilde{\varphi}_1, \\ \lambda_{hU_2} &\approx y_R \sin \epsilon \cos \varphi_4 \cos \tilde{\varphi}_1, \\ g_{WuD} = -g_{WuX} = -c_w g_{ZuU_p} &\approx \frac{g}{2} \cos \epsilon \sin \varphi_4 \cos \tilde{\varphi}_1, \end{aligned}$$

where

$$\tan \tilde{\varphi}_1 \equiv \frac{y_R f \cos \epsilon / M_1}{1 + (y_R f \sin \epsilon)^2 / M_4^2}.$$

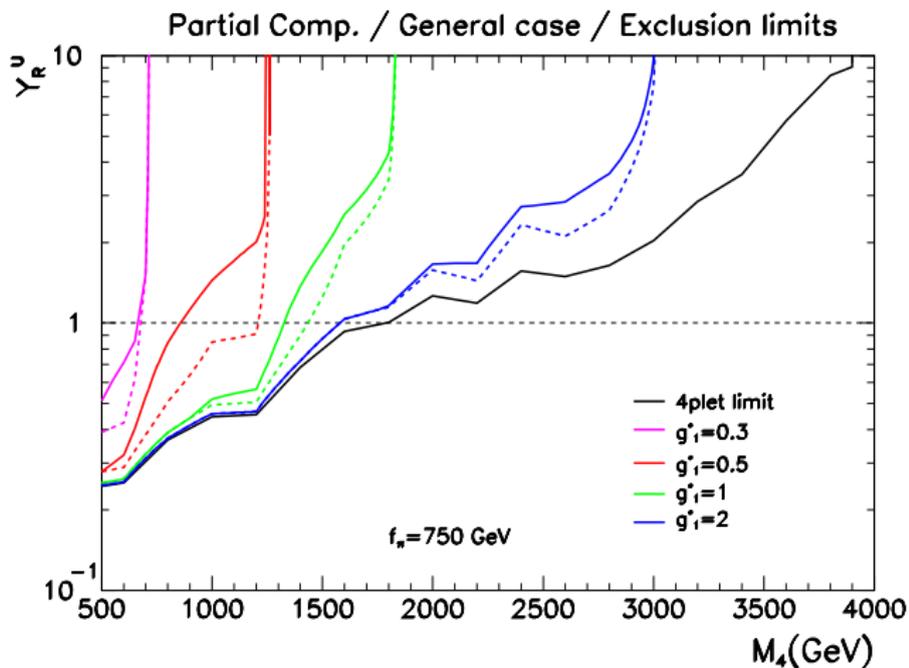
...also present: “Mixing” couplings amongst heavy quarks partners:  $\lambda_{hU_1 U_2}, g_Z U_{1/2} U_p$ , and analogous for charged couplings.

## General case: $M_1$ and $M_4$ finite.

Consequences of finite  $M_1$  for fourplet bounds:

- The single-production cross section of  $X_{5/3}, D, U_1$  is reduced.  
Physical reason: The production arises due to mixing of  $u_R$  with the fourplet, but now,  $u_R$  also mixes with the singlet.
- If the lighter up-type mass eigenstate  $U_1$  is mostly singlet (for  $M_1 \lesssim M_4$ ):  
Fourplet states  $U_p, D, X_{5/3}$  can also cascade decay via the  $U_1$   
→ The previously considered signal cross section gets reduced due to the BR into cascade decays.

# General case: $M_1$ and $M_4$ finite, up-partners



Limits on  $y_{R}^u$  as a function of  $M_4$  for different values of  $g_1^* \equiv M_1/f$ .  
 Solid: full limits. Dashed: limits ignoring signal loss due to cascade decays.

## Constraining Partners in the singlet

[TF, Jeong Han Kim, Seung Joon Lee, Sung Hak Lim, arXiv:1312.5316]

We see from the previous analysis:

- The “standard search channels” for quark partners do not yield a bound on the singlet partner because it couples to SM quarks only via the Higgs.
- A light singlet partner dilutes the bounds on the fourplet partners.

⇒ A bound on singlet partner from other search channels is required.

Two main possibilities:

- Consider the  $hhjj$  channel as an additional source of Higgs production and use “standard” Higgs search data.  
(Requires suitable observables which allow to discriminate between SM and BSM production of Higgses; e.g.  $p_T^h \leftrightarrow$  boosted signals)  
Sung Hak Lim’s talk tomorrow:  $m_{U_h} > 310 \text{ GeV}$  [TF, J.H. Kim, S.J. Lee, S.H. Lim, arXiv:1312.5316]
- Wait for ATLAS and CMS di-Higgs searches  
By now CMS published di-Higgs search results in the  $llll$  and  $ll\gamma\gamma$  channel.  
[CMS PAS HIG-13-025]  
Matching the cross-section bounds to our previous CHM analysis yields the estimate:  $m_{U_h} \gtrsim 300 \text{ GeV}$ .

## Conclusions

- Composite Higgs models provide a viable solution to the hierarchy problem and generically predict partner states to the fermions.
- The phenomenology of light quark partners strongly differs from top-partner phenomenology.
- For partially composite quarks with partners in the fourplet, we find a flavor and  $y_R$  independent bound of  $M_4^{u/c} \gtrsim 525 \text{ GeV}$  as well as stronger flavor and  $y_R$  dependent bounds ( e.g.  $M_4^u \gtrsim 1.8 \text{ TeV}$ ,  $M_4^c \gtrsim 0.8 \text{ TeV}$  for  $y_R^{u/c} = 1$ ).
- For partially composite quarks with partners in the singlet, we find a flavor- and  $y_R$  independent bound of  $M_{U_h} > 310 \text{ GeV}$  as well as increased flavor- and  $y_R$ -dependent bounds. (c.f. Sung Hak Lim's talk tomorrow:)
- We performed analogous analyses for fully composite right-handed light quarks, for which many of the aspects presented here apply as well.

## Outlook / To do

- Our analysis focussed on light quarks. Top- and bottom partners are partially but not systematically discussed elsewhere. *c.f. e.g.* [Rattazzi *et al.* (2012), Mühlleitner *et al.* (2013)]  
A more comprehensive study of bottom partners is under way.

[Jeong Han Kim, Seung Joon Lee (to appear soon)]

- Improve bounds by more detailed analysis (making use of boosted decays).

[M. Backovic, TF, S.J. Lee, J. Juknevich, (*in progress*)]

- In the current analysis, we only considered a single flavor at a time.  
→ Include full quark sector, consistent with bounds from flavor physics.

[TF, S.J. Lee, G. Perez, Y. Soreq (*in progress*)]

- On a more general level:

We only *parameterize* the lowest lying quark partner resonances.

- UV completion?
- Determination of parameters from the strong sector?
- ...

# Fully composite quarks

## Fermion embedding

Like before:

$$q_L^5 = \frac{1}{\sqrt{2}} \begin{pmatrix} id_L \\ d_L \\ iu_L \\ -u_L \\ 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} Q \\ \tilde{U} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} iD - iX_{5/3} \\ D + X_{5/3} \\ iU + iX_{2/3} \\ -U + X_{2/3} \\ \sqrt{2}\tilde{U} \end{pmatrix},$$

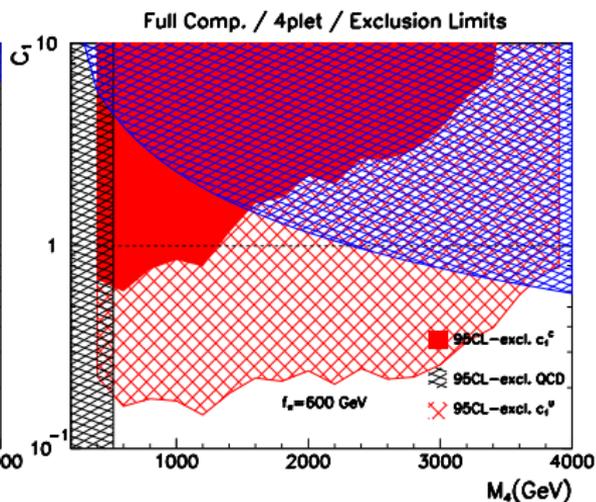
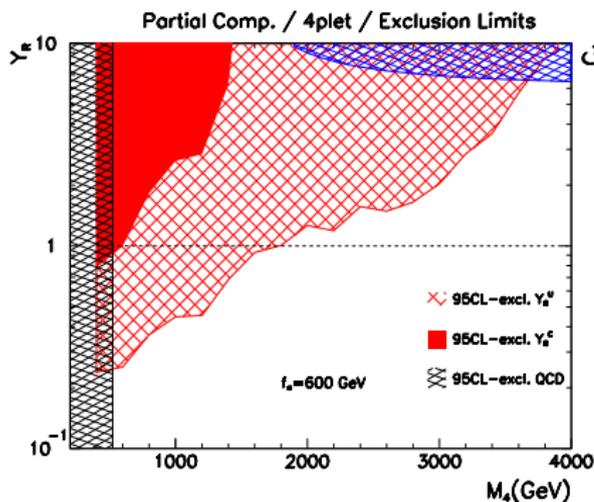
but now, embed  $u_R$  as a chiral composite  $SO(5)$  singlet.

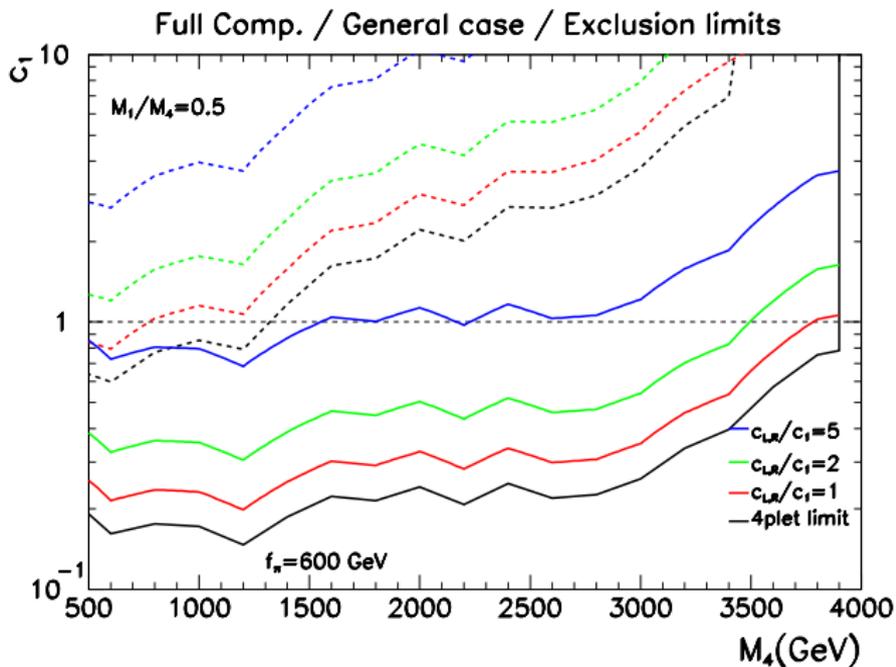
## Fermion-Lagrangian

$$\mathcal{L}_{comp}^f = i \bar{\psi} (D_\mu + ie_\mu) \gamma^\mu \psi + i \bar{u}_R \not{D} u_R - M_4 \bar{Q} Q - M_1 \bar{\tilde{U}} \tilde{U} \\ + [i c_L \bar{Q}_L^i d_\mu^i \gamma^\mu \tilde{U}_L + i c_R \bar{Q}_R^i d_\mu^i \gamma^\mu \tilde{U}_R + \text{h.c.}] + [i c_1 \bar{Q}_R^i d_\mu^i \gamma^\mu u_R + \text{h.c.}],$$

$$\mathcal{L}_{el+mix}^f = i \bar{q}_L \not{D} q_L - [y f (\bar{q}_L^5 U_{gs})_i Q_R^i + \\ + y c_2 f (\bar{q}_L^5 U_{gs})_5 u_R + y c_3 f (\bar{q}_L^5 U_{gs})_5 \tilde{U}_R + \text{h.c.}],$$

# Determining bounds from searches



General case:  $M_1$  and  $M_4$  finite, up-partners, fully composite

Limits on  $c_1^u$  (solid) and  $c_1^c$  (dashed) as a function of  $M_4$   
for different values of  $c_R/c_1$  (with  $c_L = c_R$ ).

## Constraining partner quarks in the singlet

Main difference as compared to partially composite quarks:

- The “mixed” coupling to the Higgs is naturally small for light quark partners.  
 $\Rightarrow$  QCD pair production is the dominant production process.
- The BR of  $U_h$  decays into  $W$ ,  $Z$ , and  $h$  and a light quark are  
 $\sim 50\%$ ,  $\sim 25\%$ ,  $\sim 25\%$ .  
 $\Rightarrow$  the “signal” from  $U_h \rightarrow hj \rightarrow \gamma\gamma j$  is reduced.

With an analogous study as for the partially composite quarks, we find a a flavor and  $y_R$  independent bound of  $M_{U_h} \gtrsim 212 \text{ GeV}$ .

Definition of  $d$  and  $e$  symbols:

$$d_{\mu}^i = \sqrt{2} \left( \frac{1}{f} - \frac{\sin \Pi/f}{\Pi} \right) \frac{\vec{\pi} \cdot \nabla_{\mu} \vec{\pi}}{\Pi^2} \Pi^i + \sqrt{2} \frac{\sin \Pi/f}{\Pi} \nabla_{\mu} \Pi^i$$

$$e_{\mu}^a = -A_{\mu}^a + 4i \frac{\sin^2(\Pi/2f)}{\Pi^2} \vec{\pi}^t t^a \nabla_{\mu} \vec{\pi}$$

$d_{\mu}$  symbol transforms as a fourplet under the unbroken  $SO(4)$  symmetry, while  $e_{\mu}$  belongs to the adjoint representation.

$\nabla_{\mu} \Pi$  is the "covariant derivative" of the Goldstone field  $\Pi$

$$\nabla_{\mu} \Pi^i = \partial_{\mu} \Pi^i - iA_{\mu}^a (t^a)^i_j \Pi^j,$$

$A_{\mu}$ : gauge fields of the gauged subgroup of  $SO(4) \simeq SU(2)_L \times SU(2)_R$

$$A_{\mu} = \frac{g}{\sqrt{2}} W_{\mu}^{+} (T_L^1 + iT_L^2) + \frac{g}{\sqrt{2}} W_{\mu}^{-} (T_L^1 - iT_L^2) \\ + g (c_W Z_{\mu} + s_W A_{\mu}) T_L^3 + g' (c_W A_{\mu} - s_W Z_{\mu}) T_R^3.$$

Explicit form in unitary gauge:

$$\left\{ \begin{array}{l} e_L^{1,2} = -\cos^2\left(\frac{\bar{h}}{2f}\right) W_L^{1,2} \\ e_L^3 = -\cos^2\left(\frac{\bar{h}}{2f}\right) W^3 - \sin^2\left(\frac{\bar{h}}{2f}\right) B \end{array} \right\}, \left\{ \begin{array}{l} e_R^{1,2} = -\sin^2\left(\frac{\bar{h}}{2f}\right) W_L^{1,2} \\ e_R^3 = -\cos^2\left(\frac{\bar{h}}{2f}\right) B - \sin^2\left(\frac{\bar{h}}{2f}\right) W^3 \end{array} \right\}$$

and

$$\left\{ \begin{array}{l} d_\mu^{1,2} = -\sin(\bar{h}/f) \frac{W_\mu^{1,2}}{\sqrt{2}} \\ d_\mu^3 = \sin(\bar{h}/f) \frac{B_\mu - W_\mu^3}{\sqrt{2}} \\ d_\mu^4 = \frac{\sqrt{2}}{f} \partial_\mu h, \end{array} \right. .$$