# Heterotic string phenomenology: Moduli stabilisation and non-maximally-symmetric spacetime

Stephen Angus

CTPU Journal Club
12 February 2015

# Outline

# Outline

1. **Why heterotic string theory?**
2. **Mathematical background**
   - Homology
   - Differential forms and cohomology
3. **Moduli stabilisation**
   - Moduli stabilisation overview
   - Mirror symmetry
   - Non-CY manifolds
4. **Non-maximally-symmetric spacetime**
   - Domain wall vacuum
   - Extension: cosmic strings and black holes

# Why the heterotic string?

Some reasons to study heterotic string theory:

- String theory: consistent theory of quantum gravity!
- Comes equipped with an $E_8 \times E_8$ (or SO(32)) gauge group $\Rightarrow$ good framework for grand unified models.
- Many candidate Standard Model compactifications known.
- Calabi–Yau compactification gives $\mathcal{N} = 1$ SUSY in $d = 4$ $\rightarrow$ suitable for MSSM-style models.
- Appealing mathematical framework, reasonably well-studied.

# Outline

# Homology

For a (compact) manifold $M$:

- A *p*-cycle is a $p$-dimensional submanifold with no boundary,
$$\partial c_p = 0 \ .$$
- A *p*-boundary is the boundary of a $(p+1)$-dimensional submanifold,
$$b_p = \partial d_{p+1} \ .$$
- NOTE: A boundary has no boundary $\Rightarrow$ a $p$-boundary is a $p$-cycle.

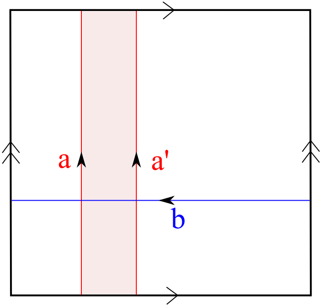
The *p*th homology group is defined as

$$H_p(M) = \frac{\{p\text{-cycles on } M\}}{\{p\text{-boundaries on } M\}} \ .$$

It classifies the $p$-cycles that are not boundaries.

# Homology example: torus $T^2$

- $a$ and $a'$ belong to the same homology class (since $a - a'$ is the boundary of the shaded region).
- $a$ and $b$ belong to different homology classes. They cannot be deformed into each other.




- Each 1-cycle can wrap the torus an integer number of times.
- The first homology group, $H_1(T^2) = \mathbb{Z} \oplus \mathbb{Z}$.

# Differential forms

- Recall that $\{\mathrm{d}x^\mu\}$ form a basis of covariant vectors.
- The wedge product is the antisymmetric tensor product, e.g.

$$\mathrm{d}x^1 \wedge \mathrm{d}x^2 = \mathrm{d}x^1 \otimes \mathrm{d}x^2 - \mathrm{d}x^2 \otimes \mathrm{d}x^1 \ .$$

- A $p$-form is a totally antisymmetric, covariant tensor of rank $p$:

$$\omega_p = \frac{1}{p!}\omega_{\mu_1 \dots \mu_p} \mathrm{d}x^{\mu_1} \wedge \dots \wedge \mathrm{d}x^{\mu_p} \ .$$

- The exterior derivative is defined as

$$\mathrm{d}\omega_p = \frac{1}{p!}\left(\partial_\rho \omega_{\mu_1 \dots \mu_p} \mathrm{d}x^\rho\right) \wedge \mathrm{d}x^{\mu_1} \wedge \dots \wedge \mathrm{d}x^{\mu_p} \ .$$

# Differential forms: example (electromagnetism)

- Define gauge potential 1-form, field strength 2-form,

$$A = A_\nu \mathrm{d}x^\nu \ , \quad F = \frac{1}{2} F_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \ .$$

- The usual relation,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ ,$$

can be written as

$$F = dA \ .$$

- Similarly, two of Maxwell's equations in tensor form,

$$\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0 \ ,$$

are now simply

$$\mathrm{d}F = 0 \ .$$

# Cohomology

- A $p$-form $\omega_p$ is closed if
$$\mathrm{d}\omega_p = 0 \; .$$
- A $p$-form $\beta_p$ is exact if there is a $(p-1)$-form $\alpha_{p-1}$ such that
$$\beta_p = \mathrm{d}\alpha_{p-1} \; .$$

Electromagnetism example:

- $\mathrm{d}F = 0 \Rightarrow F$ is closed;
- $F = \mathrm{d}A \Rightarrow F$ is also exact.

On any (compact) manifold $M$, the $p$th cohomology group is

$$H^p(M) = \frac{\{\text{closed } p\text{-forms on } M\}}{\{\text{exact } p\text{-forms on } M\}} \; .$$

It classifies the closed $p$-forms that are not exact.

# de Rham's theorem

- Homology and cohomology groups are dual to each other,

$$H_p(M) \cong H^p(M)\ .$$

- Explicitly this means that

cycles that are not boundaries $\Leftrightarrow$ closed forms that are not exact.

- Or, heuristically,

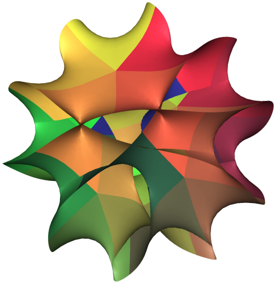
non-trivial topology $\Leftrightarrow$ non-trivial tensor structure.

# Outline

1. Why heterotic string theory?
2. Mathematical background
   - Homology
   - Differential forms and cohomology
3. **Moduli stabilisation**
   - Moduli stabilisation overview
   - Mirror symmetry
   - Non-CY manifolds
4. Non-maximally-symmetric spacetime
   - Domain wall vacuum
   - Extension: cosmic strings and black holes

# Calabi–Yau compactification

- Superstring theory is self-consistent only in 10 spacetime dimensions.
- Assume the extra 6 spatial dimensions are compactified.
- Lots of supersymmetry in $d = 10$ $\rightarrow$ want to break most of it.
- Amount of broken SUSY $\Rightarrow$ holonomy group of compactification manifold.



- Holonomy group: set of all possible rotations of a vector after transport around a closed curve on the manifold.
- Maximum holonomy is SO(6) $\cong$ SU(4) $\Rightarrow$ no SUSY preserved.
- Calabi–Yau manifold: SU(3) holonomy $\Rightarrow$ 1/4 SUSY preserved.

# What are moduli?

Compactification gives rise to scalar fields called moduli.
Three types of moduli:

- The (axio-)dilaton $S$
  $\rightarrow$ sets the string coupling, always present in string theory;
- Kähler moduli $T^i$
  $\rightarrow$ parametrise closed 2-forms $\Rightarrow$ size of 2-cycles in the geometry;
- Complex structure moduli $Z^a$
  $\rightarrow$ closed 3-forms $\Rightarrow$ 3-cycles, “shape” of compactification.

Moduli are flat directions in the potential — this is catastrophic!
Moduli need masses to prevent decompactification, 5th forces, etc.
$\rightarrow$ problem of moduli stabilisation.

# Moduli stabilisation: Type IIB example

- Example: moduli stabilisation in type IIB string theory.
- Theory contains R-R 3-form flux $F_3$ and NS-NS 3-form flux $H_3$.
- R, NS: periodicity conditions on string excitations.
- Compactify such that on the manifold, $F_3$ and $H_3$ are non-zero $\rightarrow$ flux compactification.
- In the right combination, dilaton and all complex structure moduli can be stabilised (note: 3-forms stabilise 3-cycles).
- Kähler moduli remain unstabilised, can fix with eg.
  - non-perturbative effects (KKLT),
  - non-perturbative effects and perturbative $\alpha'$ corrections (LVS).
- All moduli stabilised!

# Problems with heterotic moduli stabilisation

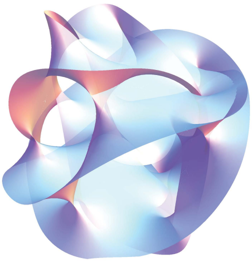
- In heterotic string theory, only have NS-NS flux $H_3$.
- Can stabilise complex structure moduli... what then?
- Dilaton can be stabilised by gaugino condensation.
- No other non-perturbative effects, no other options for flux quantisation.
- In fact, problem is even worse:

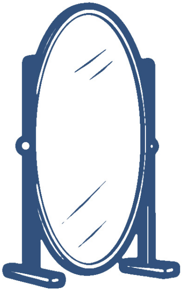
**Strominger, 1986**

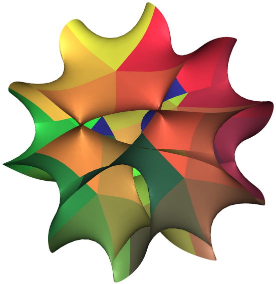
If a heterotic compactification on a manifold $Y$ has a maximally symmetric (i.e. Poincaré) vacuum and non-vanishing $H_3$, $Y$ is non-Calabi–Yau.

- Hence for a Calabi–Yau compactification, $H_3 = 0$!

# Mirror symmetry

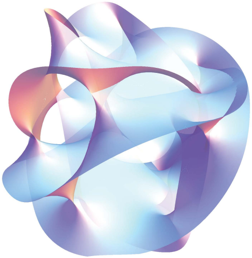


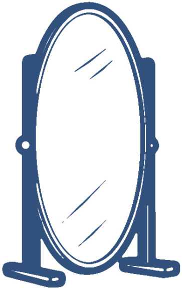


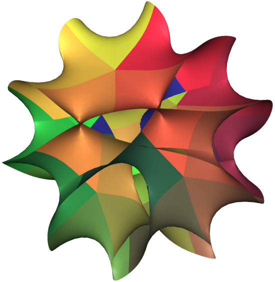


- A related issue is mirror symmetry.
- For every Calabi–Yau manifold $Y$, there is a mirror Calabi–Yau $\widetilde{Y}$.
- Mirror symmetry exchanges Kähler moduli $T^i$ and complex structure moduli $Z^a$.

# Mirror symmetry and flux compactifications







- Type IIA compactified on $Y \leftrightarrow$ type IIB compactified on $\widetilde{Y}$.
- Flux compactifications: R-R flux $F_3 \leftrightarrow F_0, F_2, F_4, F_6$.
- Problem: no obvious mirror dual for NS-NS flux $H_3$!

# What is an SU(3) structure manifold?

- Mirror dual: manifold with SU(3) structure, but not Calabi-Yau hep-th/0008142 (Vafa), hep-th/0211102 (Gurrieri, Louis, Micu, Waldram).
- SU(3) structure: there is a globally-defined spinor $\eta$ that leaves 1/4 of the SUSY unbroken.
- Calabi–Yau case: $\eta$ is covariantly constant with respect to the Levi-Civita connection $\nabla$.
- Non-CY case: $\nabla\eta \sim T^0\eta$ (note: Γ matrices/tensor indices suppressed).
- $T^0$ is the intrinsic torsion of the manifold.
- Torsion on 2-cycles can stabilise Kähler moduli $T^i$.

# A choice

There are two options:

**Option 1:**

Study the moduli space of Poincaré-invariant compactifications on SU(3)-structure manifolds.

- Has been studied in eg. hep-th/0408121 (Gurrieri, Lukas, Micu), hep-th/0507173 (de Carlos, Gurrieri, Lukas, Micu).
- Torsion quantisation understood from mirror symmetry on so-called half-flat manifolds.
- Can stabilise all moduli, but difficult to tune for consistent solutions/GUT gauge couplings. Many axions remain unstabilised.

**Option 2:**

Heterotic Calabi–Yau flux compactifications, but break maximal symmetry (Poincaré invariance) of $d = 4$ spacetime.

# Outline

# Domain wall vacuum

- Assume heterotic Calabi–Yau compactification with *H*-flux.
- Maximal symmetry in $d = 3 + 1$ broken!
- There exist 1/2-BPS domain wall solutions 1305.0594 (Klaput, Lukas, Svanes).
- 1/2-BPS: 2 of the 4 SUSY generators in $d = 4, \mathcal{N} = 1$ unbroken.
- $d = (2 + 1)$ Poincaré symmetry preserved; DW breaks symmetry in transverse $y$ direction.
- Moduli satisfy flow equations in the $y$ coordinate.

BUT…that's not realistic!

- However, Kähler moduli have not yet been stabilized explicitly.
- Doing so may "uplift" to a Minkowski vacuum (shown to happen in certain half-flat cases).

# Extension

An idea I'm currently working on, in collaboration with Eirik Svanes (LPTHE, Paris) and Cyril Matti (City University, London):

- The 1/2-BPS domain wall breaks Poincaré invariance along one coordinate.
- Mathematically, the SU(3) structure is fibred over an "interval".
- Example of an interval: the real line $\mathbb{R}$.

**Proposition:**

Fibre the SU(3) structure over an interval that is not a Cartesian coordinate direction.

Examples:

- Cylindrical $(\rho, \phi, z)$, fibre along $\rho \Rightarrow$ cosmic string;
- Spherical $(r, \theta, \phi)$, fibre along $r \Rightarrow$ black hole.

# Cosmic strings

- Cylindrical polar coordinates $(\rho, \phi, z)$: fibre along $\rho$ $\Rightarrow$ cosmic string.
- Appears to also be 1/2-BPS.
- Flow equations for moduli have the same structure as in the domain wall case.

Issues:

- That's still not realistic $\Rightarrow$ again may need to uplift to Minkowski.
- Deeper structure: relation to intersecting domain walls?
- Possible 1/4-BPS solutions?
- Work in progress!

# Black holes

- Spherical polar coordinates $(r, \theta, \phi)$: fibre along $r$
  $\Rightarrow$ "black hole" solution.
- Note: this is a toy model — just a naked singularity at the origin.
- Flow equations have analogous structure to the other 2 cases.
- Major advantage: do not necessarily need to uplift to Minkowski!

However

- Toy model does not yet account for curvature, etc.
- Not yet matched to the full 10d heterotic solution.
- Related to triple-intersecting domain walls? 1/8-BPS not possible as only 4 supercharges. . . consistency?

# Summary

- String compactifications generate moduli, which must be stabilised. This can be done using fluxes that wrap the cycles in the geometry.
- For the heterotic string, only $H_3$ present. One solution is to compactify on SU(3) structure manifolds which are not Calabi–Yau.
- An alternative is to sacrifice Poincaré invariance $\rightarrow$ domain wall solutions have been considered.
- We are currently working on extending this to cosmic strings and black holes.