

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} \lesssim 16\epsilon \left[\frac{1+a}{1+\mathcal{O}\left(\frac{a}{\epsilon^3 c_s^4}\right)} \right]$$